A HLM Guide

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Introduction

The ordinary least squares (OLS) regression often utilized in educational studies in developing predictive and explanatory models when cases/subjects are independent of each other. When this assumption is violated (e.g., subjects within a nested structure), OLS regression underestimates standard error estimates, which may lead the researcher to incorrectly conclude regression coefficient significant (Ethington, 1997). Hierarchical linear models (HLM) (also known as random coefficient models (Rosenberg, 1973), multilevel linear models (Mason et. al. 1984), or mixed linear models (Goldstein, 1986)) was developed to account for dependence among individuals within groups. HLM allows for “1) improved estimation of effects within individual units, 2) the formulation and testing of hypothesis about cross-level effects and 3) the partition of variance and covariance components among levels (Raudenbush & Bryk, 2002, p.7).” The purpose of this document is to provide a guide for users in developing a hierarchical linear model involving subjects within a nested structure (e.g., students within a school).

Engineering Education Example

Each section of this guide examines the steps in developing a hierarchical linear model utilizing the mixed linear procedure in SAS (proc mixed), SPSS (mixed), and STATA (xtmixed). To aid the process, I will explore which organizational (second level) and individual (first level) variables influence engineering students’ interactions with their professors. The dataset utilized in the example was developed for the *Engineering Change (EC2000)* project sponsored by ABET and the National Science Foundation (Grant No. EEC-9812888) and conducted by faculty members in the Center for the Study of Higher Education (CSHE) at the Pennsylvania State University (Lattuca, Terenzini, & Volkwein, 2006). This nationally representative database contains 4,461 survey responses from engineering graduates of the class of 2004 in seven engineering disciplines (aerospace, chemical, civil, computer, electrical, industrial, and mechanical) in 39 different accredited engineering institutions. The sample of colleges and universities included Doctoral, Master’s, and Bachelors’ and Specialized Institutions.

The dependent measure for this model consist of students’ responses to a 5-item scale assessing how often things occurred in their classes such as “Instructors gave me frequent feedback on my work;” “Instructors gave me detailed feedback on my work;” “Instructors guided students’ learning activities rather than lecturing or demonstrating the course material;” “I interacted with instructors as part of the course;” “I interacted with instructors outside of class (including office hours, advising, socializing, etc.)” (Lattuca, Terenzini, Volkwein, 2005). This interaction construct has an alpha reliability of .87. Appendices 1 and 2 provides a list of variables and their descriptive statistics examined for this model.

The research question for our example is to investigate which organizational characteristics and individual characteristics influence engineering student’s interactions with their professors.

Guide Notation

Table 1 provides HLM notation utilized throughout this guide. The x variables are usually associated with level-1 characteristics (e.g., students in our example) and w variables are associated with level-2 characteristics (e.g., engineering institutions in our example). Within the mixed procedure for SAS and SPSS, the user can specify the type of variable, which is the reason for differentiating between categorical (x_{cati} and w_{catk}) and continuous variables (x_{contj} and

w_{contl}), which is not a common HLM notation. For STATA, categorical variables must be coded as indicator variables; whereas in SAS and SPSS, categorical variables can either be coded as indicator variables or be kept in its original format. For example, an engineering discipline variable has the following values: aerospace, chemical, civil, computer, electrical, industrial and mechanical (seven possible values). In SAS and SPSS, an engineering discipline variable would suffice (as long as your properly classify it as categorical variable in the mixed procedure). However, for STATA, the user must choose to create six indicator variables (e.g., aerospace, chemical, civil, computer, electrical, and industrial with mechanical being the reference group), where 1 equals student is in that major and 0 equals students is not in that major. If all the indicators are set to zero, then the student would be majoring in mechanical engineering.

Table 1: HLM Notation

Y_{ij}	Dependent Measure of the ith individual within the jth organization
x_{cati}	ith categorical variable for level-1
x_{contj}	jth continuous variable for level-1
w_{catk}	kth categorical variable for level-2
w_{contl}	lth continuous variable for level-2
β_{qj}	The intercept and regression coefficients representing the effects of the level-1 independent variables on Y_{ij} in the jth organization
γ_{qs}	The intercept and regression coefficients representing the fixed effects of the level-1 and level-2 independent variables on Y_{ij}
μ_{qj}	Represents random error associated with the level-2 model with a normal distribution (N (0, τ_{qq}))
r_{ij}	Represents random error associated with the level-1 model with a normal distribution (N(0, σ^2))
$\bar{x}_{.j}$	Group mean (the variable average of all the subjects within the jth organization)
$\bar{x}_{..}$	Grand mean (the variable average of all the subjects)

The One-way ANOVA (Unconditional Model)

The unconditional model is essentially a one-way ANOVA testing whether differences exists for the dependent variables between cluster units (e.g., organizations). The general model is:

$$Y_{ij} = \mu + \alpha_j + r_{ij} \qquad (1)$$

where μ is the grand mean (the cluster unit mean and not the subject mean) of the dependent measure (Y_{ij}), α_j ~ iid $N(0, \tau_{00})$ and r_{ij} ~ iid $N(0, \sigma^2)$. α_j is the average contribution of the jth cluster unit to the dependent measure where the variability between organizations is τ_{00}. r_{ij} is the residual associated with the ith subject within the jth cluster unit. These residuals are assumed to be normally distributed with a variance of σ^2. In general, the main purpose of building statistical models is to reduce the residuals' variability (i.e., we want σ^2 to be relatively small) in order to increase the model's predictability or explanatory power. Having a large residual variability suggests that much of the differences between the cluster units is left unexplained.

The following equations, decomposes the general model (equation 1) into two levels.

Level-1 Model:

$$Y_{ij} = \beta_{0j} + r_{ij} \qquad (2)$$

The dependent value (Y_{ij}) is the measure for the ith subject within the jth cluster. r_{ij} is the residual of the ith subject within the jth cluster and is normally distributed with zero mean and a variance of σ^2 ($N(0, \sigma^2)$). σ^2 is a measure of variability among the subjects within the cluster units (i.e., subjects are nested within the organizations). β_{0j} is the intercept and the mean for the jth cluster unit and is modeled as a level-2 variable.

Level-2 Model:

$$\beta_{0j} = \gamma_{00} + \mu_{0j} \qquad (3)$$

where γ_{00} is the mean of the cluster units and μ_{0j} are the random deviations that are normally distributed with a variance of τ_{00} ($N(0, \tau_{00})$). μ_{0j} may be considered the residuals of the cluster units or the difference between a cluster unit's mean (i.e., the average of all the subjects' dependent measures within the cluster unit) and the mean of the cluster units' means. In other words, τ_{00} is the variability of the means between cluster units.

The combined model derives from substituting the level-2 model (β_{0j}) into the level-1 model's equation:

$$Y_{ij} = \gamma_{00} + \mu_{0j} + r_{ij} \qquad (4)$$

where μ_{0j} (the residuals of the cluster units) is normally distributed with zero mean and a variance of τ_{00} ($N(0, \tau_{00})$) and r_{ij} (the residuals of the subjects within the cluster units) is normally distributed with zero mean and a variance of σ^2 ($N(0, \sigma^2)$). γ_{00} corresponds to the μ and μ_{0j} to the α_j in the general model (equation1). This model can be split into two components, fixed (γ_{00}) and random (μ_{0j} and r_{ij}). The γ_{00} is fixed because it does not vary from subject to subject or cluster to cluster; whereas μ_{0j} varies from cluster unit to cluster unit and r_{ij} varies from subject to subject within cluster units.

Model Evaluation
Thus, the unconditional model partitions the variance into two components: 1) variance associated to cluster units (variance between cluster units - τ_{00}) and 2) variance associated to individuals within cluster units (variance within cluster units - σ^2). The intraclass correlation is the proportion of variance explained in the dependent measure (Y) by the clusters (the values for j) with respect to the total variance (variance between clusters and variance within cluster). The intraclass correlation ($\hat{\rho}$) is calculated as

$$\hat{\rho} = \frac{\tau_{00}}{\tau_{00}+\sigma^2} \qquad (5)$$

When examining the unconditional model, we are determining whether HLM is an appropriate model by examining the significance of τ_{00} and its relative contribution to the overall model. The intraclass correlation is a measure that examine whether the portion of variance existing between clusters (i.e., τ_{00} is large with respect to σ^2) justify utilizing HLM to account for cluster unit's effects. One reason in utilizing HLM is to account for dependencies between subjects within a cluster unit; however, if little variance is attributed to the cluster unit (i.e., either τ_{00} is not significant or $\hat{\rho}$ is small) then multiple regression is sufficient, because these methods are fairly robust when model assumptions (such as independence) are violated (Ethington, 1997). This also suggests that the majority of variance is attributed to differences between subjects with minimal influence attributed to the cluster unit.

Mixed Procedure for Unconditional Model
Identifying the random and fixed components of the model is important (Table 2), when utilizing the mixed procedure in SAS and SPSS (Figure 1). Both programs utilize a line for the user to specify the random effects. The default settings for both program has the intercept included as a fixed effect, thus, there is no need to specify it in the code (for SAS this is the "model" line and for SPSS this is the "/Fixed" line).

Table 2: Random and Fixed Components of the Unconditional Model

Model Components	Type of Variable	Interpretation
γ_{00}	Fixed	γ_{00} is the mean of the cluster units' mean.
μ_{0j}	Random	μ_{0j} is the difference between the cluster unit mean (average of the subject scores within the cluster) and the mean of the cluster units' means after accounting for the level-2 predictors. A significant variance (τ_{00}) implies that the intercepts differ between organizations.
r_{ij}	Random	r_{ij} is the residual of the subject's score after accounting for the cluster's effect (μ_{0j}).

Figure 1: Software Code for the Unconditional Model

SAS	**proc mixed** data=DATA noclprint covtest ; class Cluster Unit; model Y_{ij} = /solution; random intercept/sub= Cluster Unit;; **run**;
SPSS	MIXED Y_{ij} /CRITERIA = CIN(95) MXITER(100) MXSTEP(5) SCORING(1) SINGULAR(0.000000000001) HCONVERGE(0, ABSOLUTE) LCONVERGE(0, ABSOLUTE) PCONVERGE(0.000001, ABSOLUTE) /FIXED = | SSTYPE(3) /METHOD = REML /PRINT = SOLUTION TESTCOV /RANDOM INTERCEPT | SUBJECT(Cluster Unit;) COVTYPE(UN) .
STATA	xtmixed Y_{ij} || Cluster Unit:, reml variance

Engineering Instructor Interaction Example

For our engineering education example, we want to examine whether instructor interaction varies significantly from institution to institution and whether the proportion of this second level variance is large enough to justify the utilization of HLM. The level-1 unconditional model is shown below:

$$\text{InstructorInteraction}_{ij} = \beta_{0j} + r_{ij} \qquad (6)$$

Where j = Cal State Polytechnic, Cal State Sacromento, Case Western, … , Texas A & M, and MIT and i = 1, 2, …, $n_{institution}$. In other words, i is the ith student within the institution, where n equal the number of student responses within an institution (see Appendix 2 for the n's of each institution). The residual (r_{ij}) is normally distributed with zero mean and a variance of σ^2.

The level two model is:

$$\beta_{0j} = \gamma_{00} + \mu_{0j} \qquad (7)$$

where γ_{00} is the mean of the institutions' means on instructor interactions (i.e., average of the instructor interaction within the institution) and μ_{0j} is the deviation of the jth institution from the grand mean. μ_{0j} is assumed to be normally distributed with a variance of τ_{00} ($N(0, \tau_{00})$). In other words, τ_{00} is the variability of the instructor interaction means between the institutions.

Substituting equation 6 into 5, the complete model is

$$\text{InstructorInteraction}_{ij} = \gamma_{00} + \mu_{0j} + r_{ij} \qquad (8)$$

γ_{00} is the grand mean (i.e, the mean of the institutions' means on instructor interactions); μ_{0j} is the deviation of the jth institution; and r_{ij} is the residual of the ith student within the jth institution. Figure 2 provides the SAS, SPSS, and STATA code for this example.

Figure 2: Code for the Engineering Education Example (Unconditional Model)

<table>
<tr><td>SAS</td><td>proc mixed data=ABET noclprint covtest ;
class Institution;
model InstructorInteraction = /solution;
random intercept/sub=Institution;
run;</td></tr>
<tr><td>SPSS</td><td>MIXED
InstructorInteraction
/CRITERIA = CIN(95) MXITER(100) MXSTEP(5)
SCORING(1) SINGULAR(0.000000000001)
HCONVERGE(0, ABSOLUTE) LCONVERGE(0,
ABSOLUTE)
PCONVERGE(0.000001, ABSOLUTE)
/FIXED = | SSTYPE(3)
/METHOD = REML
/PRINT = SOLUTION TESTCOV
/RANDOM INTERCEPT | SUBJECT(Institution)
COVTYPE(UN) .</td></tr>
<tr><td>STATA</td><td>xtmixed InstructorInteraction || institution:, reml variance</td></tr>
</table>

Engineering Instructor Interaction Results

The τ_{00} and σ^2 estimates are found under the section "Covariance Parameter Estimates" for SAS[1], "Estimates of Covariance Parameters" for SPSS[2], and "Random-effects Parameters" for STATA (Figure 3). The parameter estimates from all programs are the same with τ_{00} equal to .09957 with a p-value less than .0001 (see yellow highlight in Figure 3) and σ^2 is .4194 p-value less than .0001[3] and both are significant at an alpha of .01[4]. Since τ_{00} is significant, the intercepts in the model varies from one institution to another. Consequently, this suggests that instructor interaction differ between institutions.

The intraclass correlation ($\hat{\rho}$) is .19 (.09957/ (.09957+ .4194)). Hence the proportion of variation in instructor interaction between schools is 19 percent. Since the intraclass correlation is greater than .05, HLM would be an appropriate statistical technique (Porter, 2005).

[1] If "Covariance Parameter Estimates" is not found in the output, check to see if "covtest" is included in the proc mixed statement.

[2] If "Estimates of Covariance Parameters" is not found in the output, check to see if "testcov" is included in the print line in the mixed procedure syntax.

[3] SPSS provides the p-value for a 1-tail test; divide this value by 2 to get the same value as the SAS output

[4] STATA and SPSS provide 95% confidence intervals. If the confidence interval does not include zero (0), then the variance is significant at an alpha of .05 (1-.95).

Figure 3: Estimates of Covariance Parameters for Unconditional Model

SAS Output

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z
Intercept	Institution	0.09957	0.02497	3.99	<.0001
Residual		0.4194	0.008922	47.01	<.0001

SPSS Output

Covariance Parameters

Estimates of Covariance Parameters(a)

Parameter		Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Residual		.419404	.008922	47.010	.000	.402278	.437260
Intercept [subject = Institution]	Variance	.099571	.024967	3.988	.000	.060912	.162767

a Dependent Variable: INTERACTION Interaction Scale: Stu q16k,l,m,n,o.

STATA Output

```
------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
institution: Identity        |
                  var(_cons) |   .0995705   .0249666      .0609113    .1627658
-----------------------------+------------------------------------------------
               var(Residual) |   .4194015   .0089216       .402275    .4372571
------------------------------------------------------------------------------
```

The fixed effects for the unconditional model can be found under the "Solution for Fixed Effects" in SAS output, "Estimates of Fixed Effects" in SPSS output, and "interaction" (the name of the dependent variable) in STATA output (Figure 4). The institutional mean estimate (γ_{00}) for instructor interaction is 2.3213 (p-value is .0000), which is significant at an alpha of .01.

Figure 4: Parameter Estimates for Unconditional Model

SAS Output

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	2.3213	0.05210	38	44.56	<.0001

SPSS Output

Type III Tests of Fixed Effects(a)

Source	Numerator df	Denominator df	F	Sig.
Intercept	1	35.788	1985.162	.000

a Dependent Variable: INSTRUCTORINTERACTION Interaction Scale: Stu q16k,l,m,n,o.

Estimates of Fixed Effects(a)

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Intercept	2.321300	.052099	35.788	44.555	.000	2.215616	2.426985

a Dependent Variable: INSTRUCTORINTERACTION Interaction Scale: Stu q16k,l,m,n,o.

STATA Output

```
------------------------------------------------------------------------------
 interaction |     Coef.    Std. Err.     z     P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       _cons |  2.321305  .0520993   44.56   0.000    2.219193    2.423418
------------------------------------------------------------------------------
```

The SAS and SPSS information criteria[5] output are all equal for the -2 restricted log likelihood, Akaike's information criterion (AIC), and Hurvich and Tasi's Criterion (AICC) (
Figure **5**). The Schwarz's Bayesian Criterion (BIC) is slightly different and may be to differences in the software's algorithms. The information from these fit statistics will be useful when comparing models involving instructor interaction.

Figure 5: Information Criteria

SAS Output

Fit Statistics

-2 Res Log Likelihood	8906.9
AIC (smaller is better)	8910.9
AICC (smaller is better)	8910.9
BIC (smaller is better)	8914.2

SPSS Output

Information Criteria(a)

-2 Restricted Log Likelihood	8906.914
Akaike's Information Criterion (AIC)	8910.914
Hurvich and Tsai's Criterion (AICC)	8910.917
Bozdogan's Criterion (CAIC)	8925.720
Schwarz's Bayesian Criterion (BIC)	8923.720

The information criteria are displayed in smaller-is-better forms.
a Dependent Variable: INSTRUCTORINTERACTION Interaction Scale: Stu q16k,l,m,n,o.

[5] For Stata, use the command "estat ic" after running the xtmixed command.

Including Effects of Level-2 Predictors

When examining the effects of the second level predictors, the goal is to determine whether the cluster unit characteristic (second level predictor w_{conti} and w_{cati}) has a statistical significant influence on the dependent measure (Y_{ij}). Thus, the level-1 model is the same as the unconditional model's level-1:

$$Y_{ij} = \beta_{0j} + r_{ij} \tag{9}$$

The dependent value (Y_{ij}) is the measure for the ith subject within the jth cluster. r_{ij} is the residual of the ith subject within the jth cluster and is normally distributed with zero mean and a variance of σ^2 ($N(0, \sigma^2)$). However, because cluster predictors are included the level-2 model becomes:

$$\beta_{0j} = \gamma_{00} + \gamma_{0cat1} * w_{cat1} + \ldots + \gamma_{0n} * w_{catn} + \gamma_{0cont1} * w_{cont1} + \ldots + \gamma_{0contm} * w_{contm} + \mu_{0j} \tag{10}$$

where γ_{00} is the mean of the treatments for the reference group(s) of the categorical predictor(s) and/or when the continuous predictors are equal to zero ($w_{cont1}, \ldots , w_{contm}$). The deviations or residuals of the cluster units (μ_{0j}) are random and normally distributed with a variance of τ_{00} ($N(0, \tau_{00})$). Thus, τ_{00} is the variability of the means between treatments after accounting the cluster unit's predictor variables ($w_{cat1}, \ldots, w_{catn}, w_{cont1}, \ldots , w_{contm}$).

The combined model derives from substituting the level-2 model (10) into the level-1 model's equation (9):

$$Y_{ij} = \gamma_{00} + \gamma_{0cat1} * w_{cat1} + \ldots + \gamma_{0n} * w_{catn} + \gamma_{0cont1} * w_{cont1} + \ldots + \gamma_{0contm} * w_{contm} + \mu_{0j} + r_{ij} \tag{11}$$

Model Evaluation

The "proportion reduction in variance" or "variance explained" (Raudenbush & Bryk, 2002, p. 74) measures the amount of variance explained by the level-2 predictors that is attributed to the cluster units. When creating and evaluating a model that includes level-2 predictors, the goal is to reduce the level-2 variability (τ_{00}) from the unconditional model to a model including level-2 predictors. The proportion reduction in variance is:

$$= \frac{\tau_{00}(Unconditional\ Model) - \tau_{00}(Model\ with\ Level\ 2\ predictors)}{\tau_{00}(Unconditional\ Model)} \tag{12}$$

The possible values for this measure are between one and zero, where a value of one (1) suggests that the level-2 predictors explains all the variance attributed to the cluster unit and a value of zero (0) suggests the level-2 predictors explains none of the variance attributed to the cluster unit.

The other goal when evaluating a model with level-2 predictors is to examine the significance of the variance of the cluster unit's residuals (τ_{00}). If τ_{00} is not significant, the level-2 predictors in the model explain most of the variance associated with the cluster units.

Similar to an OLS, the next step is to evaluate the significance of the level-2 predictors. The null hypothesis is that the γ_{qs} is equal to zero (H_o: $\gamma_{qs} = 0$), while the alternative hypothesis is that the γ_{qs} is not equal to zero (H_a: $\gamma_{qs} \neq 0$). If these parameter estimates ($\gamma_{0cat1}, \ldots, \gamma_{0catn}, \gamma_{0cont1}, \ldots, \gamma_{0contm}$) are significant, the associated predictors are likely to be included in the final model.

Mixed Procedure for Model with Level-2 Predictors

Identifying the random and fixed components of the model is important (Table 3), when utilizing the mixed procedure in SAS, SPSS, and STATA (Figure 6). Both programs utilize a line ("random" in SAS and "/random" in SPSS) for the user to specify the random effects; while for STATA, the random effects are specified after declaring the nested structure ("|| cluster unit;"). The default settings for all three programs has the intercept included as a fixed effect, thus, there is no need to specify it in the code (for SAS this is the "model" line and for SPSS this is the "/Fixed" line).

Table 3: Random and Fixed Components of a Model with Level-2 Predictors

Model Components	Type of Variable	Interpretation
γ_{00}	Fixed	γ_{00} is the mean of the cluster units' mean for the reference group(s) of the categorical predictor(s) and/or when the continuous predictors are equal to zero.
$\gamma_{cati0} * w_{cati}$	Fixed	γ_{cati0} is the contribution of a categorical level-2 predictor to the dependent measure. A significant γ_{cati0} implies that the level-2 predictor should be included in the final model.
$\gamma_{contj0} * w_{contj}$	Fixed	γ_{contj} is the contribution of a continuous level-2 predictor to the dependent measure. A significant γ_{contj0} implies that the level-2 predictor should be included in the final model.
μ_{0j}	Random	μ_{0j} is the difference between the cluster unit mean (average of the subject scores within the cluster) and the mean of the cluster units' means after accounting for the level-2 predictors. A significant variance (τ_{00}) implies that the intercepts differ between organizations.
r_{ij}	Random	r_{ij} is residual of the subject's score after accounting for the cluster's effect (μ_{0j}) and the level-2 predictors.

Figure 6: Software Code for Model with Only Level-2 Predictors

SAS	**proc mixed** data=DATA noclprint covtest ; class Cluster Unit w_{cat1} … w_{catn} ; model Y_{ij} = w_{cat1} … w_{catn} w_{cont1} … w_{2contm} /solution ddfm=bw; random intercept/sub= Cluster Unit; **run**;
SPSS	MIXED Y_{ij} BY w_{cat1} … w_{catn} WITH w_{cont1} … w_{2contm} /CRITERIA = CIN(95) MXITER(100) MXSTEP(5) SCORING(1) SINGULAR(0.000000000001) HCONVERGE(0, ABSOLUTE) LCONVERGE(0, ABSOLUTE) PCONVERGE(0.000001, ABSOLUTE) /FIXED = w_{cat1} … w_{catn} w_{cont1} … w_{2contm} | SSTYPE(3) /METHOD = REML /PRINT = SOLUTION TESTCOV /RANDOM INTERCEPT | SUBJECT(Cluster Unit) COVTYPE(UN) .
STATA	xtmixed Y_{ij} w_{cat1} … w_{catn} w_{cont1} … w_{2contm} || Cluster Unit:, covariance(unstructured) reml variance

Engineering Instructor Interaction Example

For our engineering education example, we want to examine which organizational characteristics (level-2 predictors) influences interactions between instructors and students between institutions. For this example, we will be examining whether an institution's Carnegie classification (research intensive, research extensive, masters' and bachelor's/other) influences instructor interactions. The level-1 model is shown below:

$$\text{InstructorInteraction}_{ij} = \beta_{0j} + r_{ij} \tag{13}$$

Where j = Cal State Polytechnic, Cal State Sacramento, Case Western, … , Texas A & M, and MIT and i = 1, 2, …, $n_{institution}$. In other words, i is the ith student within the institution, where n equal the number of student responses within an institution (see Appendix 2 for the n's of each institution). The residual (r_{ij}) is normally distributed with zero mean and a variance of σ^2.

Level-2 Model:

$$\beta_{0j} = \gamma_{00} + \gamma_{0carn\_cat} * carn\_cat + \mu_{0j} \tag{14}$$

where γ_{00} is the mean of the institutions' means on instructor interactions (i.e., average of the instructor interaction within the institution) at bachelors/others universities and μ_{0j} is the deviation of the jth institution from the conditional mean. μ_{0j} is assumed to be normally distributed with a variance of τ_{00} (N(0, τ_{00})). In other words, τ_{00} is the variability of the instructor interaction means between the institutions after accounting for the Carnegie classification[6].

[6] Since Carnegie classification is a categorical variable, the output will estimate n-1 γ_{0cat1}'s, where n is the number of levels within the categorical variable (for this example, n is 4). The user can recode a categorical variable into n-1 indicator variables. For SAS and SPSS, the default reference group is the last category; however, in SAS, the user can classify the reference group in the class statement by utilizing the following command (ref=).

Substituting equation 14 into 13, the complete model is

$$InstructorInteraction_{ij} = \gamma_{00} + \gamma_{0cat1} * carn\_cat + \mu_{0j} + r_{ij} \quad (15)$$

γ_{00} is mean of the institutions' means on instructor interactions at research institutions (which was the reference group for the carnegie classification variable); γ_{0cat1} is the contribution to the instructor's interaction score at a certain Carnegie classification, μ_{0j} is the deviation of the jth institution after accounting for the Carnegie classification; and r_{ij} is the residual of the ith student within the jth institution after accounting for the Carnegies classification. Figure 7 provides the SAS, SPSS, and STATA code for this example.

Figure 7: Code for the Engineering Education Example (With Level-2 Predictors)

SAS	**proc mixed** data=ABET noclprint covtest ; class Institution carn_cat ; model InstructorInteraction = carn_cat /solution ddfm=bw; random intercept/sub=Institution; **run**;
SPSS	MIXED InstructorInteraction BY carn_cat /CRITERIA = CIN(95) MXITER(100) MXSTEP(5) SCORING(1) SINGULAR(0.000000000001) HCONVERGE(0, ABSOLUTE) LCONVERGE(0, ABSOLUTE) PCONVERGE(0.000001, ABSOLUTE) /FIXED = carn_cat | SSTYPE(3) /METHOD = REML /PRINT = SOLUTION TESTCOV /RANDOM INTERCEPT | SUBJECT(Institution) COVTYPE(UN) .
STATA	xtmixed InstructorInteraction carn_cat_1 carn_cat_2 carn_cat_3 || institution:, covariance(unstructured) reml variance

Engineering Instructor Interaction Results

The τ_{00} and σ^2 estimates are found under the section "Covariance Parameter Estimates" for SAS[7], "Estimates of Covariance Parameters" for SPSS[8], and "Random-effects Parameters" for STATA (Figure 8). The parameter estimates from all programs are the same with τ_{00} equal to .03604 with a p-value of .0003 (see yellow highlight in Figure 8) and σ^2 is .4195 with a p-value less than .0001[9]; both are significant at an alpha of .01[10].

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[7] If "Covariance Parameter Estimates" is not found in the output, check to see if "covtest" is included in the proc mixed statement.
[8] If "Estimates of Covariance Parameters" is not found in the output, check to see if "testcov" is included in the print line in the mixed procedure syntax.
[9] SPSS provides the p-value for a 1-tail test; divide this value by 2 to get the same value as the SAS output
[10] STATA and SPSS provide 95% confidence intervals. If the confidence interval does not include zero (0), then the variance is significant at an alpha of .05 (1-.95).

The proportion of the variance explained by the institution's Carnegie classification is 64 percent ((.099571-.03604)/.099571, where .099571 is the τ_{00} of the unconditional model calculated in the previous section). This implies that 64 percent of the variance in the means of the institution's interaction between instructors and students (i.e., the group mean of the instructor interactions) can be explained by the institution's Carnegie classification. This does not imply that the Carnegie classification explains 64 percent of the variance in interaction measure for all students.

Since τ_{00} is significant, 36 percent (100-64) of the unexplained variance is associated with the cluster unit that may be described by other level-2 predictor(s).

The intraclass correlation ($\hat{\rho}$) is .079 (.03604/ (.03604+ .4195)); the proportion of variation in instructor interaction between schools having the same classification is 7.9 percent.

Figure 8: Estimates of Covariance Parameters for Model with Only Level-2 Predictors

SAS Output

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z
Intercept	Institution	0.03604	0.01054	3.42	0.0003
Residual		0.4195	0.008925	47.00	<.0001

SPSS Output

Covariance Parameters

Estimates of Covariance Parameters(a)

Parameter	Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Residual	.419497	.008925	47.001	.000	.402363	.437360
Intercept [subject = Institution] Variance	.036039	.010541	3.419	.001	.020315	.063936

a Dependent Variable: INTERACTION Interaction Scale: Stu q16k,l,m,n,o.

STATA Output

```
------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
institution: Identity        |
                  var(_cons) |  .0360398   .0105412      .020315   .0639365
-----------------------------+------------------------------------------------
               var(Residual) |  .4194937   .0089253     .4023603   .4373568
------------------------------------------------------------------------------
```

The fixed effects for the model with level-2 predictors can be found under the "Solution for Fixed Effects" in SAS output, "Estimates of Fixed Effects" in SPSS output, and "" in STATA output (Figure 9). For this example, the interpretation of the intercept (γ_{00}) is the average instructor interaction at Bachelor's institution (the reference group for the Carnegie's variable) is 2.9930. The average instructor interaction decreases .8612, .5350, and .5210 at a research extensive, research intensive, and Masters institution with respect to a Bachelor's institution. The gamma estimates (γ) were all significant at an alpha of .05.

Figure 9: Parameter Estimates for Model with Only Level-2 Predictors

SAS Output

```
                         Solution for Fixed Effects
                                 Standard
Effect       carn_cat   Estimate      Error     DF    t Value    Pr > |t|

Intercept               2.9930     0.1067     35     28.06     <.0001
carn_cat      1         -0.8162     0.1135     35     -7.19     <.0001
carn_cat      2         -0.5350     0.1633     35     -3.28     0.0024
carn_cat      3         -0.5210     0.1426     35     -3.65     0.0008
carn_cat      4            0          .         .       .         .
                   Type 3 Tests of Fixed Effects
                         Num     Den
             Effect       DF      DF    F Value    Pr > F
             carn_cat      3      35     19.05    <.0001
```

SPSS Output
Fixed Effects

Type III Tests of Fixed Effects(a)

Source	Numerator df	Denominator df	F	Sig.
Intercept	1	35.297	2748.324	.000
carn_cat	3	35.002	19.050	.000

a Dependent Variable: INTERACTION Interaction Scale: Stu q16k,l,m,n,o.

Estimates of Fixed Effects(b)

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Intercept	2.992996	.106667	36.967	28.059	.000	2.776862	3.209131
[carn_cat=1.00]	-.816240	.113511	35.931	-7.191	.000	-1.046465	-.586015
[carn_cat=2.00]	-.535025	.163291	35.564	-3.277	.002	-.866334	-.203715
[carn_cat=3.00]	-.520971	.142557	36.383	-3.654	.001	-.809985	-.231957
[carn_cat=4.00]	0(a)	0	.	.	.	.	.

a This parameter is set to zero because it is redundant.
b Dependent Variable: INSTRUCTORINTERACTION Interaction Scale: Stu q16k,l,m,n,o.

STATA Output

```
------------------------------------------------------------------------------
 instructorinteraction |     Coef.      Std. Err.     z      P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
  carn_cat_1 |          -.816231   .1135109   -7.19   0.000   -1.038708   -.5937536
  carn_cat_2 |         -.5349953   .1632911   -3.28   0.001     -.85504   -.2149507
  carn_cat_3 |         -.5209562   .1425578   -3.65   0.000   -.8003643   -.2415481
       _cons |          2.992991   .1066674   28.06   0.000    2.783927    3.202056
------------------------------------------------------------------------------
```

The SAS and SPSS information criteria[11] output are all equal for the -2 restricted log likelihood, Akaike's information criterion (AIC), and Hurvich and Tasi's Criterion (AICC) (Figure 10). The Schwarz's Bayesian Criterion (BIC) is slightly different and may be to differences in the software's algorithms. The fit statistics will be useful when comparing models involving instructor interaction (Table 4). The -2 restricted log likelihood, AIC, AICC, and BIC are smaller for the model with only level-2 predictors, which implies this model is better than the unconditional (one-way Anova).

Figure 10: Information Criteria for Model with Only Level-2 Predictors

SAS Output

Fit Statistics

-2 Res Log Likelihood	8877.5
AIC (smaller is better)	8881.5
AICC (smaller is better)	8881.5
BIC (smaller is better)	8884.8

SPSS Output

Information Criteria(a)

-2 Restricted Log Likelihood	8877.489
Akaike's Information Criterion (AIC)	8881.489
Hurvich and Tsai's Criterion (AICC)	8881.492
Bozdogan's Criterion (CAIC)	8896.294
Schwarz's Bayesian Criterion (BIC)	8894.294

The information criteria are displayed in smaller-is-better forms.
a Dependent Variable: INSTRUCTORINTERACTION Interaction Scale: Stu q16k,l,m,n,o.

Table 4: Comparing Unconditional Model and Model with only Level-2 Predictors

Information Criteria	Unconditional Model	Model with only level-2 Predictors
-2 Res Log Likelihood	8906.914	8877.489
AIC	8910.914	8881.489
AICC	8910.917	8881.492
CAIC	8925.720	8896.294
BIC	8923.720	8894.294

[11] For Stata, use the command "estat ic" after running the xtmixed command.

The Random-Coefficient Model (Level-1 Variables)

For the random-coefficient model, the researcher is examining whether the level-1 predictors have a significant influence on the dependent measure (the fixed component of the model) and whether these influences differ between cluster units (the random component of the model). Thus, the level-1 model appears very similar to a traditional OLS regression model:

Level-1 Model:

$$Y_{ij} = \beta_{0j} + \beta_{cat1j} * x_{cat1} + \ldots + \beta_{catnj} * x_{catn} + \beta_{cont1j} * x_{cont1} + \ldots + \beta_{contmj} * x_{contm} + r_{ij} \quad (16)$$

The dependent value (Y_{ij}) is the measure for the ith subject within the jth cluster unit. r_{ij} is the residual of the ith subject within the jth cluster unit and is normally distributed with zero mean and a variance of σ^2 ($N(0, \sigma^2)$) after accounting for the level-1 predictors. The β's in the level-1 model are

Level-2 Model:

$$\beta_{0j} = \gamma_{00} + \mu_{0j} \quad (17)$$
$$\beta_{cat1j} = \gamma_{cat10} + \mu_{cat1j} \quad (18)$$
$$\ldots$$
$$\beta_{catnj} = \gamma_{catn0} + \mu_{catnj} \quad (19)$$
$$\beta_{cont1j} = \gamma_{cont10} + \mu_{cont1j} \quad (20)$$
$$\ldots$$
$$\beta_{contmj} = \gamma_{cont10} + \mu_{contmj} \quad (21)$$

where the γ's ($\gamma_{00}, \gamma_{cat10}, \ldots, \gamma_{catn0}, \gamma_{cont10}, \ldots, \gamma_{cont10}$) is the average intercept/or slope across the cluster units and μ_{qj}'s (i.e., μ_{0j}, $\mu_{cat1j}, \ldots, \mu_{catnj}$, $\mu_{cont1j}, \ldots, \mu_{contmj}$) are the random deviations that are normally distributed with a variance of τ_{qq} ($N(0, \tau_{qq})$). μ_{jj}'s are interpreted as the unique contributions (or residual) of the jth cluster unit to the β's, where $\tau_{qq's}$ is the variability of these unique contributions between cluster units. This model is known as the random-coefficient model, since the coefficients may vary from cluster unit to cluster unit (i.e., the β's may have different values depending on the subject's cluster unit). In contrast, differences between cluster units are not accounted for in an OLS regression model; thus, the β's in that model do not vary from subject to subject. If the β's associated τ_{qq} is significant, then the researcher needs to model the coefficient as random.

Substituting equations 17 through 21 into equation 16, the combined model is

$$Y_{ij} = \gamma_{00} + \mu_{0j} + (\gamma_{cat10} + \mu_{cat1j}) * x_{cat1} + \ldots + (\gamma_{catn0} + \mu_{catnj}) * x_{catn} + (\gamma_{cont10} + \mu_{cont1j}) * x_{cont1} + \ldots + (\gamma_{contm0} + \mu_{contmj}) * x_{contm} + r_{ij} \quad (22)$$

$$Y_{ij} = \gamma_{00} + \gamma_{cat10} * x_{cat1} + \ldots + \gamma_{catn0} * x_{catn} + \gamma_{cont10} * x_{cont1} + \ldots + \gamma_{contm0} * x_{contm} + \mu_{cat1j} * x_{cat1} + \ldots + \mu_{catnj} * x_{catn} + \mu_{cont1j} * x_{cont1} + \ldots + \mu_{contmj} * x_{contm} + r_{ij} \quad (23)$$

Model Evaluation

The "proportion reduction in variance" or "variance explained" (Raudenbush & Bryk, 2002, p. 74) measures the amount of variance explained by the level-1 predictors. When creating and evaluating a model that includes level-1 predictors, the goal is to reduce the residual's variance in the level-1 model (σ^2) from the unconditional model to a model including level-1 predictors. The proportion reduction in variance is:

$$= \frac{\sigma^2(Unconditional\ Model) - \sigma^2(Model\ with\ Level{-}1\ predictors)}{\sigma^2(Unconditional\ Model)} \quad (24)$$

The possible values for this measure are between one and zero, where a value of one (1) suggests that the level-1 predictors explains all the variance attributed to the subject and a value of zero (0) suggests the level-1 predictors explains none of the variance attributed to the subject.

The other goal when evaluating a model with level-1 predictors is to determine whether the associated β is varying between cluster units or not. If the associated variance (τ_{qq}) is significant, then the coefficient needs to be modeled as random (e.g., equations 17, 18, 19, 20, 21). If τ_{qq} is not significant, the regression coefficient should be modeled as fixed (e.g., $\beta = \gamma$).

Similar to an OLS, the next step is to evaluate the significance of the level-1 predictors. The null hypothesis is that the γ_{qs} is equal to zero (H_o: $\gamma_{qs} = 0$), while the alternative hypothesis is that the γ_{qs} is not equal to zero (H_a: $\gamma_{qs} \neq 0$). If these parameter estimates ($\gamma_{0cat1}, \ldots, \gamma_{0catn}, \gamma_{0cont1}, \ldots, \gamma_{0contm}$) are significant, the associated predictors are likely to be included in the final model

Mixed Procedure for Unconditional Model

Table 5 lists the fixed and random components of a random-coefficient model. Identifying these components will be helpful when utilizing the mixed linear procedures for SAS, SPSS, and STATA (Figure 11).

Table 5: Random and Fixed Components of a Random-coefficient Model

Model Components	Type of Variable	Interpretation
γ_{00}	Fixed	Mean of the cluster units' mean for the reference group(s) of the categorical predictor(s) and/or when the continuous predictors are equal to zero
$\gamma_{catl0} * x_{catl}$	Fixed	Contribution of a categorical level-1 predictor to the dependent measure. A significant γ_{catl0} implies that the variable should be included in the final model.
$\gamma_{contk0} * x_{contk}$	Fixed	Contribution of a continuous level-1 predictor to the dependent measure. A significant γ_{contk0} implies that the variable should be included in the final model.
$\mu_{catlj} * x_{catl}$	Random	μ_{catlj} is the unique increment of the slope for variable x_{catl} associated with the jth cluster. The variance and its significance of this statistic is important (τ_{qq}). A significant variance implies that the slope for this variable differs significantly between organizations.

Table 3: Random and Fixed Components of a Model with Level-2 Predictors (con't)

Model Components	Type of Variable	Interpretation
$\mu_{contkj} * x_{contk}$	Random	μ_{catlj} is the unique increment of the slope for variable x_{contk} associated with the jth cluster. The variance and its significance of this statistic is important (τ_{qq}). A significant variance implies that the slope for this variable differs significantly between organizations.
μ_{0j}	Random	μ_{0j} is the difference between the cluster unit mean (average of the subject scores within the cluster) and the mean of the cluster units' means after accounting for the level-1 predictors. The variance and its significance of this statistic is more important (τ_{00}).). A significant variance implies that the intercepts for this model differs significantly between organizations.
r_{ij}	Random	Residual of the subject's score after accounting for the cluster's effect (μ_{0j}) and the level-1 predictors. The variance and its significance of this statistic is more important (σ^2).

Figure 11: Software Code for the Random-coefficient Model

SAS	**proc mixed** data=DATA noclprint covtest ; class Cluster Unit x_{cat1} … x_{catn}; model Y_{ij} = x_{cat1} … x_{catn} x_{cont1} … x_{contm} /solution ddfm=bw; random intercept x_{cat1} … x_{catn} x_{cont1} … x_{contm} /sub=Cluster Unit; **run**;
SPSS	MIXED Y_{ij} BY x_{cat1} … x_{catn} WITH x_{cont1} … x_{contm} /CRITERIA = CIN(95) MXITER(100) MXSTEP(5) SCORING(1) SINGULAR(0.000000000001) HCONVERGE(0, ABSOLUTE) LCONVERGE(0, ABSOLUTE) PCONVERGE(0.000001, ABSOLUTE) /FIXED = x_{cat1} … x_{catn} x_{cont1} … x_{contm} | SSTYPE(3) /METHOD = REML /PRINT = SOLUTION TESTCOV /RANDOM INTERCEPT x_{cat1} … x_{catn} x_{cont1} … x_{contm} | SUBJECT(Cluster Unit) COVTYPE(UN) .
STATA	xtmixed Y_{ij} x_{cat1} … x_{catn} x_{cont1} … x_{contm} || Cluster Unit: x_{cat1} … x_{catn} x_{cont1} … x_{contm} , covariance(unstructured) reml variance

Engineering Instructor Interaction Example

For the engineering education example, we will examine whether students perception of instructor clarity (clarity variable) influences their instructor interactions. For the random-coefficient model, we will also examine whether students perception of clarity differs between institutions; which would justify modeling clarity as a random coefficient model (i.e., does the $\beta_{clarityj}$ vary significantly between institutions). The level-1 model for our example is:

$$\text{InstructorInteraction}_{ij} = \beta_{0j} + \beta_{clarityj} * \text{clarity} + r_{ij} \quad (25)$$

The dependent value ($\text{InstructorInteraction}_{ij}$) is the measure for the ith subject within the jth organization (e.g., j= Cal State Polytechnics, Cal State Sacramento, Case Western, …, Texas A & M, and MIT). r_{ij} is the residual of the ith subject within the jth institution and is normally distributed with zero mean and a variance of σ^2 ($N(0, \sigma^2)$) after accounting for the level-1 predictor variable (e.g., $\beta_{clarityj}$ in our example). Since, we hypothesis that clarity may differ between institutions, the level-2 models are:

$$\beta_{0j} = \gamma_{00} + \mu_{0j} \quad (26)$$
$$\beta_{clarityj} = \gamma_{clarityj} + \mu_{clarityj} \quad (27)$$

where γ_{00} is the mean of the intercepts and μ_{0j} are the random deviations that are normally distributed with a variance of τ_{00} ($N(0, \tau_{00})$). μ_{0j} may be considered the deviations away from the mean intercepts, where τ_{00} is the variability of these means between institutions. The average slope for clarity is $\gamma_{clarity}$ and $\mu_{clarityj}$ are the unique contribution of the jth institution to the clarity slope. These contributions (or deviations) are normally distributed with a variance of τ_{11} ($N(0, \tau_{11})$). If τ_{11} is significant (i.e., the value is non-zero), this implies that the students' perception of clarity varies from institution to institution.

Substituting equations 26 and 27 into equation 25, the combined model becomes:

$$\text{InstructorInteraction}_{ij} = \gamma_{00} + \mu_{0j} + (\gamma_{clarityj} + \mu_{clarityj}) * \text{clarity} + r_{ij} \quad (28)$$
$$\text{InstructorInteraction}_{ij} = \gamma_{00} + \gamma_{clarityj} * \text{clarity} + \mu_{clarityj} * \text{clarity} + r_{ij} \quad (29)$$

From equation 28, we can see that clarity needs to be specified as a fixed ($\gamma_{clarityj}$) and a random ($\mu_{clarityj}$) effect in the mixed procedure in SAS, SPSS, and STATA (Figure 12).

Figure 12: Code for the Engineering Education Example (Random Coefficient Model)

SAS	**proc mixed** data=ABET noclprint covtest ; class Institution ; model InstructorInteraction = clarity /solution ddfm=bw; random intercept clarity/sub=Institution; **run**;
SPSS	MIXED INSTRUCTORINTERACTION WITH CLARITY /CRITERIA = CIN(95) MXITER(100) MXSTEP(5) SCORING(1) SINGULAR(0.000000000001) HCONVERGE(0, ABSOLUTE) LCONVERGE(0, ABSOLUTE) PCONVERGE(0.000001, ABSOLUTE) /FIXED = CLARITY | SSTYPE(3) /METHOD = REML /PRINT = SOLUTION TESTCOV /RANDOM INTERCEPT CLARITY | SUBJECT(Institution) COVTYPE(UN) .
STATA	xtmixed interaction clarity || institution: clarity, covariance(unstructured) reml variance

Engineering Instructor Interaction Results

The τ_{qq}'s and σ^2 estimates are found under the section "Covariance Parameter Estimates" for SAS, "Estimates of Covariance Parameters" for SPSS, and "Random-effects Parameters" for STATA (Figure 13). The parameter estimates from all statistical software programs are the same with τ_{00} equal to .009537 with a p-value of .2962 (see yellow highlight in Figure 13), τ_{11} equal to .006863 with a p-value of .0400 (see green highlight in Figure 13), and σ^2 is .3356 with a p-value less than .0001 (see grey highlight in Figure 13). τ_{11} and σ^2 are both significant at an alpha of .01. A significant σ^2 implies that other level-1predicator variables exist that may decrease the residual variance. While a significant τ_{11} suggests the slope for clarity is different between institutions (e.g., MIT students perception of clarity may differ from students at Georgia Tech).

The proportion of the variance explained by students' perception of clarity is 20 percent ((.4194-.3356)/.4194), where .4194 is the σ^2 of the unconditional model calculated in the previous section). This implies that 20 percent of the **within-institution variance** in the model can be explained by student's perception of instructor clarity. Since σ^2 is significant, 80 percent (100-20) of the unexplained within-student variance may be described by other level-1 predictors.

Figure 13: Estimates of Covariance Parameters for the Random-coefficient Model

SAS Output

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z
UN(1,1)	Institution	0.009537	0.01781	0.54	0.2962
UN(2,1)	Institution	-0.00339	0.007630	-0.44	0.6564
UN(2,2)	Institution	0.006863	0.003920	1.75	0.0400
Residual		0.3356	0.007157	46.89	<.0001

SPSS Output
Covariance Parameters

Estimates of Covariance Parameters(a)

Parameter		Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Residual		.335602	.007157	46.891	.000	.321863	.349927
Intercept + CLARITY [subject = Institution]	UN (1,1)	.009539	.017811	.536	.592	.000246	.370586
	UN (2,1)	-.003395	.007630	-.445	.656	-.018349	.011559
	UN (2,2)	.006863	.003920	1.751	.080	.002240	.021022

a Dependent Variable: INTERACTION Interaction Scale: Stu q16k,l,m,n,o.

STATA Output

```
------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
institution: Unstructured    |
                var(clarity) |   .0068621   .0039201      .0022397    .0210244
                  var(_cons) |   .0094539   .0177891      .0002366    .3778171
         cov(clarity,_cons)  |  -.0033789    .007628     -.0183295    .0115717
-----------------------------+------------------------------------------------
               var(Residual) |   .3356525   .0071582      .3219119    .3499797
------------------------------------------------------------------------------
```

The fixed effects for the random-coefficient model can be found under the “Solution for Fixed Effects” in SAS output, “Estimates of Fixed Effects” in SPSS output, and “Interaction” in STATA output (Figure 14). For this example, the interpretation of the intercept (γ_{00}) is the average instructor interaction is .6080, when the clarity score is zero. The instructor interaction increases .5407 for every one point increases in student’s perception of clarity. The fixed effect for clarity ($\gamma_{clarityj}$) is significant (p-values less than .0001) at an alpha of .05, which implies that this variable should be kept for the complete model (following section).

Figure 14: Parameter Estimates for Model with the Random-coefficient Model

SAS Output

```
                         Solution for Fixed Effects
                     Standard
Effect      Estimate     Error    DF   t Value   Pr > |t|
Intercept     0.6080   0.05200    38     11.69    <.0001
CLARITY       0.5407   0.02052  4421     26.36    <.0001

              Type 3 Tests of Fixed Effects
                 Num    Den
Effect            DF     DF   F Value   Pr > F

CLARITY            1   4421    694.67   <.0001
```

SPSS Output

Fixed Effects

Type III Tests of Fixed Effects(a)

Source	Numerator df	Denominator df	F	Sig.
Intercept	1	32.079	136.681	.000
CLARITY	1	29.753	694.670	.000

a Dependent Variable: INSTRUCTORINTERACTION Interaction Scale: Stu q16k,l,m,n,o.

Estimates of Fixed Effects(a)

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Intercept	.607987	.052004	32.079	11.691	.000	.502068	.713907
CLARITY	.540712	.020515	29.753	26.357	.000	.498800	.582624

a Dependent Variable: INSTRUCTORINTERACTION Interaction Scale: Stu q16k,l,m,n,o.

STATA Output

```
------------------------------------------------------------------------------
 interaction |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     clarity |   .5404191   .0205099    26.35   0.000     .5002205    .5806178
       _cons |   .6088723   .0519638    11.72   0.000     .5070252    .7107194
------------------------------------------------------------------------------
```

The SAS and SPSS information criteria[12] output are all equal for the -2 restricted log likelihood, Akaike's information criterion (AIC), and Hurvich and Tasi's Criterion (AICC) (Figure 15). The Schwarz's Bayesian Criterion (BIC) is slightly different and may be to differences in the software's algorithms. The fit statistics will be useful when comparing models involving instructor interaction (Table 6). The -2 restricted log likelihood, AIC, AICC, and BIC are smallest for the random-coefficient model, which implies this model is better than the previously examined models.

Figure 15: Information Criteria for the Random-coefficient Model

SAS Output

Fit Statistics

-2 Res Log Likelihood	7910.7
AIC (smaller is better)	7918.7
AICC (smaller is better)	7918.7
BIC (smaller is better)	7925.4

SPSS Output

Information Criteria(a)

-2 Restricted Log Likelihood	7910.716
Akaike's Information Criterion (AIC)	7918.716
Hurvich and Tsai's Criterion (AICC)	7918.725
Bozdogan's Criterion (CAIC)	7948.327
Schwarz's Bayesian Criterion (BIC)	7944.327

The information criteria are displayed in smaller-is-better forms.
a Dependent Variable: INSTRUCTORINTERACTION Interaction Scale: Stu q16k,l,m,n,o.

Table 6: Comparing Unconditional Model and Model with only Level-2 Predictors

Information Criteria	Unconditional Model	Model with only level-2 Predictors	Model with only level-1 Predictors
-2 Res Log Likelihood	8906.914	8877.489	7910.716
AIC	8910.914	8881.489	7918.716
AICC	8910.917	8881.492	7918.725
CAIC	8925.720	8896.294	7948.327
BIC	8923.720	8894.294	7944.327

[12] For Stata, use the command "estat ic" after running the xtmixed command.

An Intercepts- and Slopes-as-Outcomes Model (Complete Model)

Now that we have identify the level-1 and level-2 predictors, the intercepts- and slopes-as-outcomes model combines the findings from the model with level-2 predictors and the random-coefficients model to build a complete explanatory model, which accounts for the variability within and between cluster units. The level-1 model for the intercepts- and slopes-as outcomes model (Equation 29) is the same as the random-coefficient model (Equation 15)

Level-1 Model:

$$Y_{ij} = \beta_{0j} + \beta_{cat1j} * x_{cat1} + \ldots + \beta_{catnj} * x_{catn} + \beta_{cont1j} * x_{cont1} + \ldots + \beta_{contmj} * x_{contm} + r_{ij} \quad (30)$$

The dependent value (Y_{ij}) is the measure for the ith subject within the jth cluster unit. r_{ij} is the residual of the ith subject within the jth cluster unit and is normally distributed with zero mean and a variance of σ^2 ($N(0, \sigma^2)$) after accounting for the level-1 predictors. The β's in the level-1 model are similar to the random-coefficient model, except all slopes and the intercept will include the significant level-2 predictors found in the model with level-2 predictors (Equations 30-34).

Level-2 Model:

$$\beta_{0j} = \gamma_{00} + \gamma_{0cat1} * w_{cat1} + \ldots + \gamma_{0catn} * w_{catn} + \gamma_{0cont1} * w_{cont1} + \ldots + \gamma_{0contm} * w_{contm} + \mu_{0j} \quad (31)$$

$$\beta_{cat1j} = \gamma_{cat10} + \gamma_{cat1cat1} * w_{cat1} + \ldots + \gamma_{cat1n} * w_{catn} + \gamma_{cat1cont1} * w_{cont1} + \ldots + \gamma_{cat1contm} * w_{contm} + \mu_{cat1j} \quad (32)$$

…

$$\beta_{catnj} = \gamma_{catn0} + \gamma_{catncat1} * w_{cat1} + \ldots + \gamma_{catncatn} * w_{catn} + \gamma_{catn1cont1} * w_{cont1} + \ldots + \gamma_{catncontm} * w_{contm} + \mu_{catnj} \quad (33)$$

$$\beta_{cont1j} = \gamma_{cont10} + \gamma_{cont1cat1} * w_{cat1} + \ldots + \gamma_{cont1catn} * w_{catn} + \gamma_{cont1cont1} * w_{cont1} + \ldots + \gamma_{cont1contm} * w_{contm} + \mu_{cont1j} \quad (34)$$

…

$$\beta_{contmj} = \gamma_{contm0} + \gamma_{contmcat1} * w_{cat1} + \ldots + \gamma_{contmcatn} * w_{catn} + \gamma_{contmcont1} * w_{cont1} + \ldots + \gamma_{contmcontm} * w_{contm} + \mu_{contmj} \quad (35)$$

where the γ's ($\gamma_{00}, \gamma_{cat10}, \ldots, \gamma_{catn0}, \gamma_{cont10}, \ldots, \gamma_{cont10}$) is the average intercept/or slope across the cluster units and μ_{qj}'s (i.e., μ_{0j}, μ_{cat1j}, … , μ_{catnj}, μ_{cont1j}, … , μ_{contmj}) are the random deviations that are normally distributed with a variance of τ_{qq} ($N(0, \tau_{qq})$) after accounting for the level-2 predictors. Substituting Equations 31, 32, 33, 34, 35, into equations leads to the combined model (Equation 36).

Combined Model:

$$\begin{aligned} Y_{ij} &= \gamma_{00} + \gamma_{0cat1} * w_{cat1} + \ldots + \gamma_{0catn} * w_{catn} + \gamma_{0cont1} * w_{cont1} + \ldots + \gamma_{0contm} * w_{contm} + \mu_{0j} \\ &+ (\gamma_{cat10} + \gamma_{cat1cat1} * w_{cat1} + \ldots + \gamma_{cat1n} * w_{catn} + \gamma_{cat1cont1} * w_{cont1} + \ldots + \gamma_{cat1contm} * w_{contm} + \mu_{cat1j}) * x_{cat1} \\ &+ \ldots \\ &+ (\gamma_{catl0} + \gamma_{catlcat1} * w_{cat1} + \ldots + \gamma_{catlcatn} * w_{catn} + \gamma_{catlcont1} * w_{cont1} + \ldots + \gamma_{catlcontm} * w_{contm} + \mu_{catlj}) * x_{catn} \\ &+ (\gamma_{cont10} + \gamma_{cont1cat1} * w_{cat1} + \ldots + \gamma_{cont1n} * w_{catn} + \gamma_{cont1cont1} * w_{cont1} + \ldots + \gamma_{cont1contm} * w_{contm} + \mu_{cont1j}) * x_{cont1} \\ &+ \ldots \\ &+ (\gamma_{contk0} + \gamma_{contkcat1} * w_{cat1} + \ldots + \gamma_{contkn} * w_{catn} + \gamma_{contkcont1} * w_{cont1} + \ldots + \gamma_{contkcontm} * w_{contm} + \mu_{contkj}) * x_{contk} \\ &+ r_{ij} \end{aligned} \quad (36)$$

Model Evaluation

As seen in the model with only level-2 predictors and the random-coefficients model, the "proportion reduction in variance" or "variance explained" evaluates how much variance the predictors account for in the level-1 and level-2 models. The number of proportion reduction in variance equations (Equations 37, 38, and 39) equals the number of random variables in the model (e.g., μ_{0j}, μ_{cat1j}, …, μ_{catnj}, μ_{cont1j}, …, μ_{contmj} and r_{ij}).

$$\text{Within Cluster Unit Variance Explained} = \frac{\sigma^2(Unconditional\ Model) - \sigma^2(Complete\ Model)}{\sigma^2(Unconditional\ Model)} \qquad (37)$$

$$\text{Between Cluster Unit Variance Explained} = \frac{\tau_{00}\ (Unconditional\ Model) - \tau_{00}\ (Complete\ Model)}{\tau_{00}\ (Unconditional\ Model)} \qquad (38)$$

$$\text{Varying Slope Variance Explained} = \frac{\tau_{qq}\ (Random{-}coefficients\ Model) - \tau_{qq}\ (Complete\ Model)}{\tau_{qq}\ (Random{-}coefficients\ Model)} \qquad (39)$$

The possible values for this measure are between one and zero, where a value of one (1) suggests that the predictor(s) explain all the variance attributed to the subject for the level-1 model and to the cluster unit for the level-2 models (i.e., the β's). A value of zero implies that the predictor(s) explain none of the variance attributed to the subject for the level-1 model and to the cluster unit for the level-2 models (i.e., the β's).

As seen in the random-coefficients model, level-1 predictors are evaluated to determine whether the associated β is varying between cluster units or not. If the associated variance (τ_{qq}) is significant, then the coefficient needs to be modeled as random (e.g., equations 31, 32, 33, 34 35). If τ_{qq} is not significant, the regression coefficient should be modeled as fixed (e.g., $\beta = \gamma$).

The next step is to evaluate the significance of the predictors. The null hypothesis is that the γ_{qs} is equal to zero (H_o: $\gamma_{qs} = 0$), while the alternative hypothesis is that the γ_{qs} is not equal to zero (H_a: $\gamma_{qs} \neq 0$). If these parameter estimates (γ_{0cat1}, …, γ_{0catn}, γ_{0cont1}, … , γ_{0contm}) are significant, these predictors needs to be included in the final model; all insignificant predictors should be removed from the final model.

Mixed Procedure for Unconditional Model

Table 7 lists the fixed and random components of a random-coefficient model. Identifying these components will be helpful when utilizing the mixed linear procedures for SAS, SPSS, and STATA (Figure 19).

Table 7: Random and Fixed Components of a Complete Model

Model Components	Type of Variable	Interpretation
γ_{00}	Fixed	γ_{00} is the mean of the cluster units' mean for the reference group(s) of the categorical predictor(s) and/or when the continuous predictors are equal to zero.
$\gamma_{0cati} * w_{cati}$	Fixed	γ_{0cati} is the contribution of a categorical level-2 predictor to the dependent measure. A significant γ_{0cati} implies that the variable should be included in the final model.
$\gamma_{0contj} * w_{contj}$	Fixed	γ_{0contj} is the contribution of a continuous level-2 predictor to the dependent measure. A significant γ_{0contj} implies that the variable should be included in the final model.
$\gamma_{catk0} * x_{catk}$	Fixed	γ_{catk0} is the contribution of a categorical level-1 predictor to the dependent measure. A significant γ_{catk0} implies that the variable should be included in the final model.
$\gamma_{contl0} * x_{contl}$	Fixed	$\gamma_{contl0}$is the contribution of a categorical level-1 predictor to the dependent measure. A significant γ_{contl0} implies that the variable should be included in the final model.
$\gamma_{catkcati} * w_{cati} * x_{catk}$	Fixed	$\gamma_{caticatk}$ is the interaction effect between a categorical level-1 and a categorical level-2 predictor on the dependent measure. A significant $\gamma_{caticatk}$ implies that the variable should be included in the final model.
$\gamma_{catkcontj} * w_{contj} * x_{catk}$	Fixed	$\gamma_{catkcontj}$ is the interaction effect between a categorical level-1 and a continuous level-2 predictor on the dependent measure. A significant $\gamma_{catkcontj}$ implies that the variable should be included in the final model.
$\gamma_{contlcati} * w_{cati} * x_{contl}$	Fixed	$\gamma_{contlcati}$ is the interaction effect between a continuous level-1 and a continuous level-2 predictor on the dependent measure. A significant $\gamma_{contlcati}$ implies that the variable should be included in the final model.
$\gamma_{contlcontj} * w_{contj} * x_{contl}$	Fixed	$\gamma_{contlcontj}$ is the interaction effect between a categorical level-1 and a categorical level-2 predictor on the dependent measure. A significant $\gamma_{contlcontj}$ implies that the variable should be included in the final model.
$\mu_{catij} * x_{cati}$	Random	μ_{catij} is the unique increment of the slope for variable x_{cati} associated with the jth cluster. The variance and its significance of this statistic is important (τ_{qq}). A significant variance implies that the slope for this variable differs significantly between organizations.

Table 5: Random and Fixed Components of a Random-coefficient Model (con't)

Model Components	Type of Variable	Interpretation
$\mu_{contjj} * x_{contj}$	Random	μ_{contjj} is the unique increment of the slope for variable x_{contj} associated with the jth cluster. The variance and its significance of this statistic is important (τ_{qq}). A significant variance implies that the slope for this variable differs significantly between organizations.
μ_{0j}	Random	μ_{0j} is the difference between the cluster unit mean (average of the subject scores within the cluster) and the mean of the cluster units' means after accounting for all the predictors in the model. A significant variance (τ_{00}) implies that the intercepts differ between organizations.
r_{ij}	Random	r_{ij} is residual of the subject's score after accounting for the cluster's effect (μ_{0j}) and the all the predictors in the model.

Figure 16: Software Code for the Complete Model

SAS	**proc mixed** data=DATA noclprint covtest ; class Cluster Unit x_{cat1} … x_{catl} w_{cat1} … w_{catn}; model Y_{ij} = x_{cat1} … x_{catn} w_{cat1} … w_{catn} x_{cont1} … x_{contk} w_{cont1} … w_{2contm} (all possible interactions between first and second level variables) /solution ddfm=bw; random intercept x_{cat1} … x_{catl} x_{cont1} … x_{contk} /sub= Cluster Unit; **run**;
SPSS	MIXED Y_{ij} BY x_{cat1} … x_{catn} w_{cat1} … w_{catn} WITH x_{cont1} … x_{contk} w_{cont1} … w_{2contm} /CRITERIA = CIN(95) MXITER(100) MXSTEP(5) SCORING(1) SINGULAR(0.000000000001) HCONVERGE(0, ABSOLUTE) LCONVERGE(0, ABSOLUTE) PCONVERGE(0.000001, ABSOLUTE) /FIXED = x_{cat1} … x_{catl} w_{cat1} …w_{catn} x_{cont1} … x_{contm} w_{cont1} … w_{2contm} (all possible interactions between first and second level variables) | SSTYPE(3) /METHOD = REML /PRINT = SOLUTION TESTCOV /RANDOM INTERCEPT x_{cat1} … x_{catl} x_{cont1} … x_{contk} | SUBJECT (Cluster Unit) COVTYPE(UN) .
STATA	xtmixed interaction x_{cat1} … x_{catl} w_{cat1} … w_{catn} x_{cont1} … x_{contk} w_{cont1} … w_{2contm} (all possible interactions between first and second level variables) || Cluster Unit: x_{cat1} … x_{catl} x_{cont1} … x_{contk} covariance(unstructured) reml variance

Engineering Instructor Interaction Example

The complete HLM model for our engineering education example will examine institutional and student characteristics that influence student's interaction with his/her professor. The institutional predictor variable included in this model is the organization's Carnegie Classification (variable examined in the model with only level-2 predictors) and the student characteristics include high school grade point average (q10bround), student's perception of instructor clarity (clarity), the amount of perceived collaboration in the program (collab), and the student's perception of program openness (prog_open). Appendix 1 and Appendix 2 provides the descriptive statistics of these variables.

In our investigation of random-coefficients model, the perception of clarity differ significantly between institutions; thus for the complete model it will be modeled as a random slope. When analyzing program openness (analysis not shown here), this predictor differ significantly between institutions with a significant slope effect. A student's high school grade point average, and perceived collaboration in the program did not differ significantly between institutions, but the slope effects are significant (analysis not shown here); thus these level-1 predictors are modeled as fixed effects.

The level-1 model for our engineering example is

$$\begin{aligned}\text{InstructorInteraction}_{ij} = {} & \beta_{0j} + \beta_{clarityj} * \text{clarity} + \beta_{prog\_openj} * \text{prog\_open} \\ & + \beta_{q10bround} * \text{q10bround} + \beta_{collab} * \text{collab} \\ & + r_{ij}\end{aligned} \tag{40}$$

The dependent value ($\text{InstructorInteraction}_{ij}$) is the measure for the ith subject within the jth organization (e.g., j= Cal State Polytechnics, Cal State Sacramento, Case Western, …, Texas A & M, and MIT). r_{ij} is the residual of the ith subject within the jth institution and is normally distributed with zero mean and a variance of σ^2 ($N(0, \sigma^2)$) after accounting for the level-1 predictor variable (e.g., $\beta_{clarityj}$ in our example). Since, we hypothesis that the intercept, clarity, and program openness may differ between institutions, the level-2 models are:

$$\beta_{0j} = \gamma_{00} + \gamma_{0carn\_cat} * \text{carn\_cat} + \mu_{0j} \tag{41}$$
$$\beta_{clarityj} = \gamma_{clarity0} + \gamma_{claritycarn\_cat} * \text{carn\_cat} + \mu_{clarityj} \tag{42}$$
$$\beta_{prog\_openj} = \gamma_{prog\_open0} + \gamma_{prog\_opencarn\_cat} * \text{carn\_cat} + \mu_{prog\_openj} \tag{43}$$

γ_{00} is the mean of the intercepts and μ_{qj}'s are the random deviations that are normally distributed with a variance of τ_{00} ($N(0, \tau_{00})$) after accounting for Carnegie Classification. μ_{0j} may be considered the deviations away from the mean intercepts, where τ_{00} is the variability of these means between institutions. The average slope for clarity is $\gamma_{clarity}$ and $\mu_{clarityj}$ are the unique contribution of the jth institution to the clarity slope after accounting for the institution's Carnegie classification. These contributions (or deviations) are normally distributed with a variance of τ_{11} ($N(0, \tau_{11})$). If τ_{11} is significant (i.e., the value is non-zero), this implies that the students' perception of clarity varies from institution to institution. The average slope for program openness is γ_{prog_open0} and μ_{prog_openj} are the unique contribution of the jth institution to the program openess slope after accounting for the institution's Carnegie classification. These contributions (or deviations) are normally distributed with a variance of τ_{22} ($N(0, \tau_{22})$). If τ_{22} is significant (i.e., the value is non-zero), this implies that the students' perception of program openness varies from institution to institution.

After substituting equations 41, 42, and 43 into equation 40, the complete model becomes

$$InstructorInteraction_{ij} = \gamma_{00} + \gamma_{0carn\_cat} * carn\_cat + \mu_{0j} + (\gamma_{clarity0} + \gamma_{claritycarn\_cat} * carn\_cat + \mu_{clarityj}) * clarity + (\gamma_{prog\_open0} + \gamma_{prog\_opencarn\_cat} * carn\_cat + \mu_{prog\_openj}) * prog\_open + \beta_{q10bround} * q10bround + \beta_{collab} * collab + r_{ij} \quad (44)$$

$$InstructorInteraction_{ij} = \gamma_{00} + \gamma_{0carn\_cat} * carn\_cat + \mu_{0j} + \gamma_{clarity0} * clarity + \gamma_{claritycarn\_cat} * carn\_cat * clarity + \mu_{clarityj} * clarity + \gamma_{prog\_open0} * prog\_open + \gamma_{prog\_opencarn\_cat} * carn\_cat * prog\_open + \mu_{prog\_openj} * prog\_open + \beta_{q10bround} * q10bround + \beta_{collab} * collab + r_{ij} \quad (45)$$

The random components of the complete model are the intercept (μ_{0j}), clarity ($\mu_{clarityj}$ * clarity), program openness (μ_{prog_openj} * prog_open), and the residuals (r_{ij}). The fixed components are Carnegie Classification (γ_{0carn_cat} * carn_cat), clarity ($\gamma_{clarity0}$ * clarity), interaction between clarity and Carnegie Classification ($\gamma_{claritycarn\_cat}$ * carn_cat * clarity), program openness (γ_{prog_open0} *prog_open), interaction between program openness and Carnegie Classification ($\gamma_{prog\_opencarn\_cat}$ * carn_cat * prog_open), high school grade point average ($\beta_{q10bround}$ * q10bround), and coloration (β_{collab} * collab). Figure 17 is the SAS code for the Equation 45 model.

Figure 17: SAS Code for Complete Model Include Interaction Terms

```
proc mixed data=ABET noclprint covtest noitprint;
      class Institution carn_cat;
      model InstructorInteraction = q10bround clarity collab prog_open
carn_cat carn_cat*clarity carn_cat*prog_open /solution ddfm=bw;
      random intercept clarity prog_open /sub=Institution type=simple;
run;
```

Figure 18: Solution for Fixed Effects of Complete Model

Solution for Fixed Effects

Effect	carn_cat	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept		-0.03434	0.2400	35	-0.14	0.8871
q10bround		0.02403	0.007187	4412	3.34	0.0008
CLARITY		0.4052	0.07011	4412	5.78	<.0001
COLLAB		0.2785	0.01232	4412	22.61	<.0001
PROG_OPEN		0.2122	0.04390	4412	4.83	<.0001
carn_cat	1	-0.3738	0.2431	35	-1.54	0.1331
carn_cat	2	-0.3090	0.3035	35	-1.02	0.3157
carn_cat	3	-0.2036	0.2904	35	-0.70	0.4878
carn_cat	4	0	.	.	.	.
CLARITY*carn_cat	1	-0.04829	0.07196	4412	-0.67	0.5022
CLARITY*carn_cat	2	-0.05499	0.09208	4412	-0.60	0.5504
CLARITY*carn_cat	3	0.000864	0.08804	4412	0.01	0.9922
CLARITY*carn_cat	4	0	.	.	.	.
PROG_OPEN*carn_cat	1	0.02418	0.04528	4412	0.53	0.5935
PROG_OPEN*carn_cat	2	0.09291	0.05868	4412	1.58	0.1134
PROG_OPEN*carn_cat	3	-0.02885	0.05626	4412	-0.51	0.6081
PROG_OPEN*carn_cat	4	0	.	.	.	.

When examining the fixed effects for the model, the interaction terms between the level-1 and level-2 variables are not significant (p-values for the clarity * carn_cat and prog_open* carn_cat are all greater than .05). This implies that even though clarity and program openness may vary between institutions, the institution's Carnegie Classification does influence student's perception of clarity and program openness significantly on instructor interaction (the dependent measure). Removing the interaction terms creates a more parsimonious model (Equation 46). Figure 19 is the SAS, SPSS, and STATA code for our complete model with Equations 47, 48, and 49 represent the new level-2 models.

$$\begin{aligned} InstructorInteraction_{ij} = {} & \gamma_{00} + \gamma_{0carn\_cat} * carn\_cat + \mu_{0j} + \gamma_{clarity0} * clarity \\ & + \mu_{clarityj} * clarity + \gamma_{prog\_open0} * prog\_open \\ & + \mu_{prog\_openj} * prog\_open + \beta_{q10bround} * q10bround + \\ & \beta_{collab} * collab + r_{ij} \end{aligned} \qquad (46)$$

$$\beta_{0j} = \gamma_{00} + \gamma_{0carn\_cat} * carn\_cat + \mu_{0j} \qquad (47)$$
$$\beta_{clarityj} = \gamma_{clarity0} + \mu_{clarityj} \qquad (48)$$
$$\beta_{prog\_openj} = \gamma_{prog\_open0} + \mu_{prog\_openj} \qquad (49)$$

Figure 19: Code for the Engineering Education Example (Complete Model)

SAS	**proc mixed** data=ABET noclprint covtest noitprint; class Institution carn_cat; model InstructorInteraction = q10bround clarity collab prog_open carn_cat /solution ddfm=bw; random intercept clarity prog_open /sub=Institution type=un; **run**;
SPSS	MIXED INSTRUCTORINTERACTION BY carn_cat WITH CLARITY PROG_OPEN q10bround COLLAB /CRITERIA = CIN(95) MXITER(100) MXSTEP(5) SCORING(1) SINGULAR(0.000000000001) HCONVERGE(0, ABSOLUTE) LCONVERGE(0, ABSOLUTE) PCONVERGE(0.000001, ABSOLUTE) /FIXED = q10bround CLARITY COLLAB PROG_OPEN carn_cat | SSTYPE(3) /METHOD = REML /PRINT = SOLUTION TESTCOV /RANDOM INTERCEPT CLARITY PROG_OPEN | SUBJECT(Institution) COVTYPE(UN) .
STATA	xtmixed interaction q10bround clarity collab prog_open carn_cat_1 carn_cat_2 carn_cat_3|| institution: clarity prog_open, covariance(unstructured) reml variance

Engineering Instructor Interaction Results

The τ_{qq}'s and σ^2 estimates are found under the section "Covariance Parameter Estimates" for SAS[13], "Estimates of Covariance Parameters" for SPSS[14], and "Random-effects Parameters" for STATA (Figure 20). The parameter estimates from all statistical software programs are similar with τ_{11} (intercept variance) equal to .00231 with a p-value of .0912 (see yellow highlight in Figure 20), τ_{22} (clarity slope variance) equal to .005045 with a p-value of .0440 (green highlight in Figure 20), τ_{33} (program openness slope) equal to .0747 with a p-value of .0747 (blue highlight in Figure 20) and σ^2 is .2435 with a p-value less than .0001[15]. All variances except for program openness are significant at an alpha of .05[16]. A significant σ^2 implies that other level-1 predicator variables exist that may decrease the residual variance. While a significant τ_{22} suggests the slope for clarity may be explained by other level-2 predictor variables.

Some researchers may decide that the slope for program openness does not vary between institutions since its variance is not significant at an alpha of .05; thus, they may decide to model this variable as fixed effect (the following section shows that the fixed effect is significant). I decided to keep this variable as a random effect because it is significant at an alpha of .1; however, decisions such as these should be guided by theory instead of arbitrary statistical rules.

The proportion of the variance explained by student's characteristics (clarity, program openness, high school grade point average, and collaboration) is 41.94 ((.4194-.2435)/.4194), where .4194 is the σ^2 of the unconditional model). This implies that 41.94 percent of the **within-institution variance** in the model is explained by student's perception of instructor clarity, program openness, high school grade point average, and collaboration. Since σ^2 is significant, 58.06 percent (100-41.94) of the unexplained within-student variance may be described by other level-1 predictors.

The proportion of the variance explained by the institution's Carnegie classification is 78 percent ((.099571-.022341)/.099571), where .099571 is the τ_{00} of the unconditional model calculated in the previous section). This implies that 78 percent of the **between-institution variance** is explained by the institution's Carnegie classification.

With no predictor variables for the slope coefficients (Equations 48 and 49), the proportion of the variance is not calculated for clarity and program openness slope.

---

[13] If "Covariance Parameter Estimates" is not found in the output, check to see if "covtest" is included in the proc mixed statement.

[14] If "Estimates of Covariance Parameters" is not found in the output, check to see if "testcov" is included in the print line in the mixed procedure syntax.

[15] SPSS provides the p-value for a 1-tail test; divide this value by 2 to get the same value as the SAS output

[16] STATA and SPSS provide 95% confidence intervals. If the confidence interval does not include zero (0), then the variance is significant at an alpha of .05 (1-.95).

Figure 20: Estimates of Covariance Parameters for Complete Model

SAS Output

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z
UN(1,1)	Institution	0.02231	0.01673	1.33	0.0912
UN(2,1)	Institution	-0.00799	0.006420	-1.25	0.2131
UN(2,2)	Institution	0.005045	0.002958	1.71	0.0440
UN(3,1)	Institution	-0.00064	0.002912	-0.22	0.8273
UN(3,2)	Institution	-0.00074	0.001330	-0.55	0.5799
UN(3,3)	Institution	0.001353	0.000939	1.44	0.0747
Residual		0.2435	0.005213	46.72	<.0001

SPSS Output

Covariance Parameters

Estimates of Covariance Parameters(a)

Parameter		Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Residual		.243546	.005213	46.719	.000	.233540	.253981
Intercept + CLARITY + PROG_OPEN [subject = Institution]	UN (1,1)	.022314	.016731	1.334	.182	.005133	.097009
	UN (2,1)	-.007993	.006420	-1.245	.213	-.020575	.004590
	UN (2,2)	.005045	.002958	1.706	.088	.001599	.015919
	UN (3,1)	-.000636	.002912	-.218	.827	-.006343	.005072
	UN (3,2)	-.000736	.001330	-.554	.580	-.003342	.001870
	UN (3,3)	.001354	.000939	1.442	.149	.000348	.005272

a Dependent Variable: INTERACTION Interaction Scale: Stu q16k,l,m,n,o.

STATA Output

Random-effects Parameters	Estimate	Std. Err.	[95% Conf.	Interval]
institution: Unstructured				
var(clarity)	.0050387	.0029566	.0015953	.0159142
var(prog_o~n)	.0013536	.0009383	.0003479	.0052667
var(_cons)	.0223133	.0167345	.0051307	.0970393
cov(clarity,prog_o~n)	-.0007326	.00133	-.0033393	.0018741
cov(clarity,_cons)	-.0079833	.0064177	-.0205618	.0045951
cov(prog_o~n,_cons)	-.0006456	.0029154	-.0063597	.0050686
var(Residual)	.2435884	.0052139	.2335806	.2540249

The fixed effects for the complete model can be found under the "Solution for Fixed Effects" in SAS output, "Estimates of Fixed Effects" in SPSS output, and "Interaction" in STATA output (Figure 21). All the fixed effects are significant at an alpha of .05 (p-values for all predictor variables are less than .05); thus all variables are kept in the final model. For this example, the interpretation of the intercept (γ_{00}) is the average instructor interaction is .0007987 for institutions with the Bachelor's Carnegie Classification when all the continuous variables are set equal to zero. The instructor interaction increases .3662 for every one point increases in student's perception of clarity after controlling for the other predictor variables. The largest influence on instructor interaction is the school's Carnegie Classification, which decreases .4450 when the school is a research extensive when compared to a Bachelor's institution, holding other variables constant.

Figure 21: Parameter Estimates for Complete Model

SAS Output

Solution for Fixed Effects

Effect	carn_cat	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept		0.007987	0.09893	35	0.08	0.9361
q10bround		0.02425	0.007176	4418	3.38	0.0007
CLARITY		0.3662	0.01860	4418	19.68	<.0001
COLLAB		0.2796	0.01231	4418	22.72	<.0001
PROG_OPEN		0.2359	0.01192	4418	19.79	<.0001
carn_cat	1	-0.4450	0.07972	35	-5.58	<.0001
carn_cat	2	-0.2442	0.1090	35	-2.24	0.0315
carn_cat	3	-0.2724	0.09754	35	-2.79	0.0084
carn_cat	4	0	.	.	.	.

Type 3 Tests of Fixed Effects

Effect	Num DF	Den DF	F Value	Pr > F
q10bround	1	4418	11.42	0.0007
CLARITY	1	4418	387.44	<.0001
COLLAB	1	4418	516.20	<.0001
PROG_OPEN	1	4418	391.73	<.0001
carn_cat	3	35	12.81	<.0001

Figure 21: Parameter Estimates for Complete Model (con't)

SPSS Output
Fixed Effects

Type III Tests of Fixed Effects(a)

Source	Numerator df	Denominator df	F	Sig.
Intercept	1	113.807	12.281	.001
q10bround	1	4430.797	11.418	.001
CLARITY	1	30.754	387.435	.000
COLLAB	1	4414.609	516.199	.000
PROG_OPEN	1	43.537	391.713	.000
carn_cat	3	40.175	12.814	.000

a Dependent Variable: INSTRUCTORINTERACTION Interaction Scale: Stu q16k,l,m,n,o.

Estimates of Fixed Effects(b)

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Intercept	.007990	.098931	110.005	.081	.936	-.188069	.204048
q10bround	.024247	.007176	4430.797	3.379	.001	.010179	.038315
CLARITY	.366184	.018604	30.754	19.683	.000	.328229	.404139
COLLAB	.279613	.012307	4414.609	22.720	.000	.255485	.303740
PROG_OPEN	.235901	.011919	43.537	19.792	.000	.211873	.259930
[carn_cat=1.00]	-.445046	.079720	47.368	-5.583	.000	-.605390	-.284702
[carn_cat=2.00]	-.244238	.109044	43.071	-2.240	.030	-.464135	-.024341
[carn_cat=3.00]	-.272435	.097540	46.709	-2.793	.008	-.468693	-.076177
[carn_cat=4.00]	0(a)	0	.	.	.	.	.

a This parameter is set to zero because it is redundant.
b Dependent Variable: INSTRUCTORINTERACTION Interaction Scale: Stu q16k,l,m,n,o.

STATA Output

```
------------------------------------------------------------------------------
 interaction   |     Coef.     Std. Err.     z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
   q10bround |   .024235   .0071764     3.38    0.001     .0101696    .0383004
     clarity    |  .3658772   .0185964    19.67   0.000     .3294289    .4023255
      collab     |  .2796545   .0123085    22.72    0.000     .2555302    .3037788
   prog_open  |  .2358166   .0119179    19.79    0.000     .2124579    .2591754
  carn_cat_1  | -.4449726   .0797291    -5.58    0.000    -.6012387   -.2887065
  carn_cat_2  | -.2441312   .1090524    -2.24    0.025      -.45787   -.0303923
  carn_cat_3  |   -.272286   .0975487    -2.79    0.005    -.4634779   -.0810941
       _cons     |  .0088802   .0989372     0.09    0.928    -.1850331    .2027936
------------------------------------------------------------------------------
```

The SAS and SPSS information criteria[17] output are all equal for the -2 restricted log likelihood, Akaike's information criterion (AIC), and Hurvich and Tasi's Criterion (AICC) (Figure 22). The

[17] For Stata, use the command "estat ic" after running the xtmixed command.

Schwarz's Bayesian Criterion (BIC) is slightly different and may be to differences in the software's algorithms. The fit statistics will be useful when comparing models involving instructor interaction (Table 4). The -2 restricted log likelihood, AIC, AICC, and BIC are smallest for the complete, which implies the complete model is best when compared to other models examined.

Figure 22: Information Criteria for the Complete Model

SAS Output

Fit Statistics

-2 Res Log Likelihood	6495.2
AIC (smaller is better)	6509.2
AICC (smaller is better)	6509.2
BIC (smaller is better)	6520.9

SPSS Output

Information Criteria(a)

-2 Restricted Log Likelihood	6495.212
Akaike's Information Criterion (AIC)	6509.212
Hurvich and Tsai's Criterion (AICC)	6509.237
Bozdogan's Criterion (CAIC)	6561.021
Schwarz's Bayesian Criterion (BIC)	6554.021

The information criteria are displayed in smaller-is-better forms.
a Dependent Variable: INTERACTION Interaction Scale: Stu q16k,l,m,n,o.

Table 8: Comparing Unconditional Model and Model with only Level-2 Predictors

Information Criteria	Unconditional Model	Model with only level-2 Predictors	Model with only level-1 Predictors	Complete Model
-2 Res Log Likelihood	8906.914	8877.489	7910.716	6495.212
AIC	8910.914	8881.489	7918.716	6509.212
AICC	8910.917	8881.492	7918.725	6509.237
CAIC	8925.720	8896.294	7948.327	6561.021
BIC	8923.720	8894.294	7944.327	6554.021

Comparing Variance Structures

Besides estimating the regression coefficients, the HLM method also estimates the second level model variances and the covariance between the second level models (i.e., the β's). For β_{0j} , we want to examine whether the variance (τ_{00}) is significant between cluster units (e.g., j=cluster unit 1, cluster unit 2, …) for the intercept. If the HLM model includes random slopes (β_{ij} , where i = 1, 2, …. and j=cluster unit 1, cluster 2, …), the covariance among the random slopes and random intercept are estimated (τ_{01} , τ_{02} , …). Examining the covariance between random coefficients provides insight on the relationship between variables at the second level of the model.

Most researchers are inclined to estimate all the variances and covariance among the random coefficients, hence specifying the unstructured covariance structure in their HLM procedure (Figure 23). When the model becomes complex, the number of variance-covariance parameters estimated increases by 2n for every new random level-1 predictor added to the model, which will increase computational processing time. Thus, imposing a covariance structure (e.g., simple/ diagonal, compound symmetry) may be necessary. In developing your HLM model, you may also notice that estimating the covariance is unnecessary, because they are not significant. This suggests that another covariance structure (such as simple) may be sufficient and may even improve your model's fit statistics. Figure 23 defines common covariance structures for HLM models involving subjects nested in organizational structures (e.g., Raudenbush and Bryk's (2002) math achievement model) and Figure 24 provides the software code to specify the covariance structures in SAS, SPSS and STATA. Figure 25 displays the SAS code for the instructor interaction example specifying each of these structures (see green highlight). Readers should know that other covariance structures exists for personal growth models (see Singer's (1998) opposite naming task) such as autoregressive and toeplitz.

Figure 23: Covariance Structures

Type	Matrix Structure	Assumption
Unstructured	$\begin{bmatrix} \sigma_1^2 & \sigma_{21} & \sigma_{31} \\ \sigma_{21} & \sigma_2^2 & \sigma_{32} \\ \sigma_{31} & \sigma_{32} & \sigma_3^2 \end{bmatrix}$	All variances and covariance among the random intercept and random slopes assumed to be different. HLM procedure estimates n *n -1 parameters, where n is the number of random coefficients (random intercepts plus number of random slopes) in the model.
Simple/Diagonal	$\begin{bmatrix} \sigma_1^2 & 0 & 0 \\ 0 & \sigma_2^2 & 0 \\ 0 & 0 & \sigma_3^2 \end{bmatrix}$	Only n random coefficients variances are estimated by the HLM. This structure assumes the covariance among the random coefficients is zero.
Compound Symmetry	$\begin{bmatrix} \sigma^2+\sigma_1 & \sigma_1 & \sigma_1 \\ \sigma_1 & \sigma^2+\sigma_1 & \sigma_1 \\ \sigma_1 & \sigma_1 & \sigma^2+\sigma_1 \end{bmatrix}$	HLM procedure estimates two (2) parameters and assumes the random coefficients' variances are equal and the covariance among the random coefficients is equal.

Figure 24: Software Code for the Complete Model

Software	Unstructured	Simple	Compound Symmetry
SAS	type = un	type = simple	type =cs
SPSS	COVTYPE(UN)	COVTYPE(DIAG)	COVTYPE(CS)
STATA	covariance(unstructured)	covariance(independent)	covariance(exchangeable)

Figure 25: SAS Software Code for Different Covariance Structures in SAS for Instructor Interaction Example

Unstructured	**proc mixed** data=ABET noclprint covtest noitprint; class Institution carn_cat; model InstructorInteraction = q10bround clarity collab prog_open carn_cat /solution ddfm=bw; random intercept clarity prog_open /sub=Institution type=un; **run**;
Simple/Diagonal	**proc mixed** data=ABET noclprint covtest noitprint; class Institution carn_cat; model InstructorInteraction = q10bround clarity collab prog_open carn_cat /solution ddfm=bw; random intercept clarity prog_open /sub=Institution type=simple; **run**;
Compound Symmetry	**proc mixed** data=ABET noclprint covtest noitprint; class Institution carn_cat; model InstructorInteraction = q10bround clarity collab prog_open carn_cat /solution ddfm=bw; random intercept clarity prog_open /sub=Institution type=cs; **run**;

Results

The fit statistics determine the best model (Figure 26), when comparing models. We see that the compound symmetry covariance structure is best since the AIC, AICC, and BIC are smallest when compared to the simple/diagonal and unstructured model.

Figure 26: Information Criteria

Unstructured Covariance Structure

Fit Statistics

-2 Res Log Likelihood	6495.2
AIC (smaller is better)	6509.2
AICC (smaller is better)	6509.2
BIC (smaller is better)	6520.9

Simple/Diagonal Covariance Structure

Fit Statistics

-2 Res Log Likelihood	6500.2
AIC (smaller is better)	6508.2
AICC (smaller is better)	6508.2
BIC (smaller is better)	6514.8

Compound Symmetry Covariance Structure

Fit Statistics

-2 Res Log Likelihood	6499.3
AIC (smaller is better)	6505.3
AICC (smaller is better)	6505.3
BIC (smaller is better)	6510.3

Figure 27 is the covariance parameter estimates of the different models. When examining the unstructured covariance structure model, we see that all three covariance are not significant at an alpha of .05 (p-values are .2131 for UN(2,1), .8273 for UN(3,1) and .5799 for UN(3,2)). In other words, I fail to reject the null hypothesis (H_o: covariance is equal to zero) and conclude that the covariance may be zero. This suggests that a simple/diagonal covariance structure may be more appropriate for this model, which assumes that the covariance among the random coefficients (in this example, the intercept, clarity, and program openness) is zero. Assuming a zero covariance, suggests that the random coefficients are independent of each other, i.e., no relationships exists between the random intercept and the random slopes. For this model, if a negative covariance between the clarity and program openness slope is significant (which it is not in this case), then as the slope clarity increases, the slope for program openness would decrease. If a positive covariance exists, then as the slope of clarity increases, the slope for program openness would also increase. Thus, even though the compound symmetry model has a better-fit statistic than simple/diagonal, the model may not be appropriate since the covariance for this model is not significant at an alpha of .05 (CS p-value = .3179).

Figure 27: Estimates of Covariance Parameters

Unstructured Covariance Structure

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z
UN(1,1)	Institution	0.02231	0.01673	1.33	0.0912
UN(2,1)	Institution	-0.00799	0.006420	-1.25	0.2131
UN(2,2)	Institution	0.005045	0.002958	1.71	0.0440
UN(3,1)	Institution	-0.00064	0.002912	-0.22	0.8273
UN(3,2)	Institution	-0.00074	0.001330	-0.55	0.5799
UN(3,3)	Institution	0.001353	0.000939	1.44	0.0747
Residual		0.2435	0.005213	46.72	<.0001

Simple/Diagonal Covariance Structure

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z
Intercept	Institution	0.001538	0.004759	0.32	0.3732
CLARITY	Institution	0.001139	0.000586	1.95	0.0259
PROG_OPEN	Institution	0.000626	0.000457	1.37	0.0853
Residual		0.2446	0.005220	46.86	<.0001

Compound Symmetry Covariance Structure

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z
Variance	Institution	0.002418	0.001571	1.54	0.0619
CS	Institution	-0.00061	0.000613	-1.00	0.3179
Residual		0.2443	0.005218	46.81	<.0001

The compound symmetry model also assumes that the variances between the second level models are equal (i.e., $\tau_{00} = \tau_{11} = \tau_{22}$). However, from the unstructured covariance structure estimates, we see that the variances between the random coefficients appear to be different (UN(1,1) = intercept variance = .2231; UN(2,2) = clarity's slope variance = .005045; and UN(3,3) = program openness' slope variance = .006045). This suggests that the compound symmetry model may not be appropriate. The simple/diagonal variance estimates for intercept variance (τ_{00}), clarity's slope variance (τ_{11}), and program openness' slope variance (τ_{22}) are .001538, 001139, and .000626. Even though, the estimates appear to be different, the significance of these parameters are consistent (i.e., only clarity is significant).

For all three models, the residual variance estimates, the σ^2 for the HLM model, are relatively the same (.2435, .2446, and .2443 – see blue highlight in Figure 27). The unstructured covariance structure may be best because the residual variance is the smallest; however, nothing is gained or lost when choosing between the three types of covariance structures.

When examining the regression coefficient estimates (i.e., the γ's in the HLM model) between the three covariance structure models (Table 9, see Figure 28 for complete output), only the intercept estimates appear to be different (.007987, .03469, and .02716). The estimates for the intercept vary due to the restrictions placed on the covariance (i.e., set to zero for simple or all covariance between parameters equal in compound symmetry). Selecting a covariance matrix structure appears to have little influence on estimating the regression coefficients.

Table 9: Comparison of Regression Coefficient between Covariance Structures

	Unstructured	Simple/Diagonal	Compound Symmetry
Intercept	.007987	.03469	.02716
Q10bround	.02425	.02447	.02424
Clarity	.3662	.3618	.3614
Collaboration	.2796	.2788	.2788
Program Openness	.2359	.2357	.2384
Carnegie Classification (1)	-.4450	-.4554	-.4542
Carnegie Classification (2)	-.2442	-.2661	-.2699
Carnegie Classification (3)	-.2724	-.2734	-.2689
Carnegie Classification (4)	0	0	0

For this model, I would mostly likely utilize the simple/diagonal covariance structure, because of the smaller fit statistics (AIC, AICC, BIC) compared to the unstructured model and the covariance among random coefficients are not significant. The compound symmetry does not appear appropriate because of the variances among the random coefficients do not appear to be equal and the covariance between the random coefficients are not significant.

Figure 28: Parameter Estimates for Model

Unstructured Covariance Structure

```
                    Solution for Fixed Effects
                           Standard
     Effect      carn_cat   Estimate     Error     DF    t Value   Pr > |t|
     Intercept            0.007987    0.09893     35      0.08     0.9361
     q10bround              0.02425   0.007176   4418      3.38     0.0007
     CLARITY                 0.3662    0.01860   4418     19.68     <.0001
     COLLAB                  0.2796    0.01231   4418     22.72     <.0001
     PROG_OPEN               0.2359    0.01192   4418     19.79     <.0001
     carn_cat      1        -0.4450    0.07972     35     -5.58     <.0001
     carn_cat      2        -0.2442    0.1090      35     -2.24     0.0315
     carn_cat      3        -0.2724    0.09754     35     -2.79     0.0084
     carn_cat      4              0          .      .         .          .
                    Type 3 Tests of Fixed Effects
                          Num     Den
             Effect        DF      DF    F Value    Pr > F
             q10bround      1    4418     11.42    0.0007
             CLARITY        1    4418    387.44    <.0001
             COLLAB         1    4418    516.20    <.0001
             PROG_OPEN      1    4418    391.73    <.0001
             carn_cat       3      35     12.81    <.0001
```

Simple/Diagonal Covariance Structure

Solution for Fixed Effects

Effect	carn_cat	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept		0.03469	0.09694	35	0.36	0.7226
q10bround		0.02447	0.007181	4418	3.41	0.0007
CLARITY		0.3618	0.01489	4418	24.29	<.0001
COLLAB		0.2788	0.01231	4418	22.65	<.0001
PROG_OPEN		0.2357	0.01083	4418	21.77	<.0001
carn_cat	1	-0.4554	0.08321	35	-5.47	<.0001
carn_cat	2	-0.2661	0.1141	35	-2.33	0.0256
carn_cat	3	-0.2734	0.1021	35	-2.68	0.0112
carn_cat	4	0	.	.	.	.

Type 3 Tests of Fixed Effects

Effect	Num DF	Den DF	F Value	Pr > F
q10bround	1	4418	11.61	0.0007
CLARITY	1	4418	590.20	<.0001
COLLAB	1	4418	512.83	<.0001
PROG_OPEN	1	4418	474.02	<.0001
carn_cat	3	35	12.10	<.0001

Compound Symmetry Covariance Structure

Solution for Fixed Effects

Effect	carn_cat	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept		0.02716	0.09469	35	0.29	0.7759
q10bround		0.02424	0.007176	4418	3.38	0.0007
CLARITY		0.3614	0.01563	4418	23.12	<.0001
COLLAB		0.2788	0.01230	4418	22.66	<.0001
PROG_OPEN		0.2384	0.01247	4418	19.13	<.0001
carn_cat	1	-0.4542	0.07940	35	-5.72	<.0001
carn_cat	2	-0.2699	0.1076	35	-2.51	0.0169
carn_cat	3	-0.2689	0.09700	35	-2.77	0.0089
carn_cat	4	0	.	.	.	.

Type 3 Tests of Fixed Effects

Effect	Num DF	Den DF	F Value	Pr > F
q10bround	1	4418	11.41	0.0007
CLARITY	1	4418	534.41	<.0001
COLLAB	1	4418	513.35	<.0001
PROG_OPEN	1	4418	365.82	<.0001
carn_cat	3	35	13.44	<.0001

Centering

Raudenbush and Bryk (2002) offer three suggestions in the location of the level-1 predictor variables: the natural X metric (also refer to as the raw score), centering around the grand mean, and centering around the group mean (also refer to as centering within context). Utilizing our engineering example, Equation 49 is an example utilizing the natural X metric for our model. The natural score maintains the raw measure (i.e., no transformations applied to the variable's values).

$$\text{InstructorInteraction}_{ij} = \beta_{0j} + \beta_{\text{clarity}j} * \text{clarity} + r_{ij} \quad (49)$$

Grand mean centering involves calculating the mean of all the subjects within the study and subtracting that value from the original measure. This procedure would center the mean to equal to zero. For our engineering example, I would calculate the students' mean on their perception of clarity and then subtract that value from the original clarity measure.

$$\text{InstructorInteraction}_{ij} = \beta_{0j} + \beta_{\text{clarity}j} * (\text{clarity} - \widehat{clarity}_{..}) + r_{ij} \quad (50)$$

Centering around the group mean entails calculating the mean of the subjects score within a cluster unit and then subtracting that value from all the subjects within that group (Equation 50). For our example, we found the mean clarity score within an institution (e.g., Georgia Tech) and then subtract that value from all Georgia Tech students' clarity score. This would be done for the other 38 institutions in our study.

$$\text{InstructorInteraction}_{ij} = \beta_{0j} + \beta_{\text{clarity}j} * (\text{clarity} - \widehat{clarity}_{.j}) + r_{ij} \quad (51)$$

Centering (either around the grand or the group) versus maintaining the natural metric provides a more meaningful interpretation of the fixed intercept term (γ_{00}) (Singer, 1997); however, Kreft, de Leeuw, and Aiken (1995) conclude that there is no statistically correct choice in where the level-1 predictor is located. They found that the natural X metric and the grand mean location produce equivalent statistical models (i.e., models that have the same expectations and dispersions). Even though the regression coefficients between the two models may be different, an equivalent model implies a one-to-one transformation exist so that model can be converted to the other. The advantage of centering (either grand mean or group mean) though does remove a large portion of the confounding slope and intercept variance (Kreft, de Leeuw, & Aiken, 1995). Statistically, utilizing a group-mean centering provides a different model from the natural X metric and grand mean centering; thus, the decision in centering should depend on the researcher's theoretical model. The suggestion though is to center (either grand mean or group mean) the variable to provide a more meaningful interpretation of the fixed intercept term and to ease estimation computations and stability (Kreft, de Leeuw, & Aiken, 1995).

Contextual Models

Contextual models include the group mean as a variable in the level-2 model (Burstein, 1980). This implies the predictor has a within- and between effect on the dependent measure. An example is Raudenbush and Bryk's (2002) math achievement example, where they include the school district's average socio economic status at the second level. The implication is that not only does a student's socio economic status (within- effect), but also the relative wealth of the student's school district (between-effect), measured by the mean socio economic status within the school district, influences his/her match achievement. Including the group mean as a level-2 predictor provides contextualizes the individual's situation. The choice of centering within the level-1 model influences the interpretation of the regression coefficients. Group-mean centering decomposes the effects into within- and between cluster units; while grand mean centering provides the compositional (the sum of the within – and between cluster unit effects) and the within- cluster effect (see Table 5.11 on p. 140 in Raudenbush and Bryk (2002)).

The decision to include the group mean as a level-2 predictor depends on the researcher's theoretical model in whether the variable's context influences the dependent measure (Kreft, de Leeuw, & Aiken, 1995). For example, when developing a contextual HLM models for student success, SAT scores are often included as a level-1 predictor. The inclusion of the SAT group mean as a level-2 predictor depends on whether the researcher also believes success is a school effect (Kreft, de Leeuw, & Aiken, 1995). The school effect is the belief that having peers of similar ability also influences student's success. As noted above, including the group mean as a level-2 predictor partitions the variable's effects into within- and between cluster units; thus, the researcher in developing a contextual model must be able to justify that the predictor would have a between cluster effect.

Centering in SAS, SPSS, and STATA

Unlike HLM6, the mixed linear procedures in SAS, SPSS, and STATA do not offer an option of centering a variable. If you want to center a variable, the center variable must be created. The grand mean centered variable is computed by calculating the variable's mean and then creating a new variable (e.g., GrandMeanCenterVariable) by subtracting the grand mean from the original variable.

My suggestion in creating a group-mean centered variable is first calculate the group-mean for the variable (SAS use proc means, SPSS use the MEANS procedure, and STATA use "tab (variable) summarize(cluster unit)"). Once the group-mean is tabulated, create a group-mean variable (GroupMeanVariable), which will be useful, if you decide to include this variable as a level-2 predictor. Figure 29 provides one method of creating the GroupMeanVariable. Then create a group-mean centered variable (e.g., GroupMeanCenterVariable) by subtracting the group mean variable from the original variable.

Figure 29: Creating a Group Mean Variable in SPSS

```
RECODE
 ClusterUnit
  (ClusterUnit1=Cluster Unit 1's Mean)
  (ClusterUnit2=Cluster Unit 2's Mean)
  …
  (ClusterUnit j=Cluster Unit j's Mean)
 INTO  GroupMeanVariable .
EXECUTE .
```

Conclusion

Figure 30 provides a guide in building an HLM. There is no statistical reasoning to the order of these steps. Unlike OLS regression, the mixed procedure does not have a method option (e.g., stepwise, forward or backward) in creating a parsimonious model with only significant terms. Thus, the first step in identifying possible variables for an HLM is to develop an OLS regression model with only significant terms (Step 1). Since, OLS regression underestimates the size of the standard errors of the regression coefficients; thus, significant terms in the OLS model may be insignificant in the HLM and non-significant terms in the OLS regression model will be non-significant terms in HLM.

The next step (Step 2) is to evaluate the unconditional model, which partitions the variance into between and within cluster units. With the variance partition in this fashion, the intraclass correlation (ρ) is calculated (Step 3). The within and between variances will be used as baselines when evaluating predictors and the completeness of the final model (e.g., proportion reduction in variance measures). You may decide that the OLS regression model suffices, because the intraclass correlation is small enough (5%); that accounting for organizational characteristics is not worthwhile. However, if the intraclass correlation is large enough, then Step 4 is creating a model with only level-2 predictors.

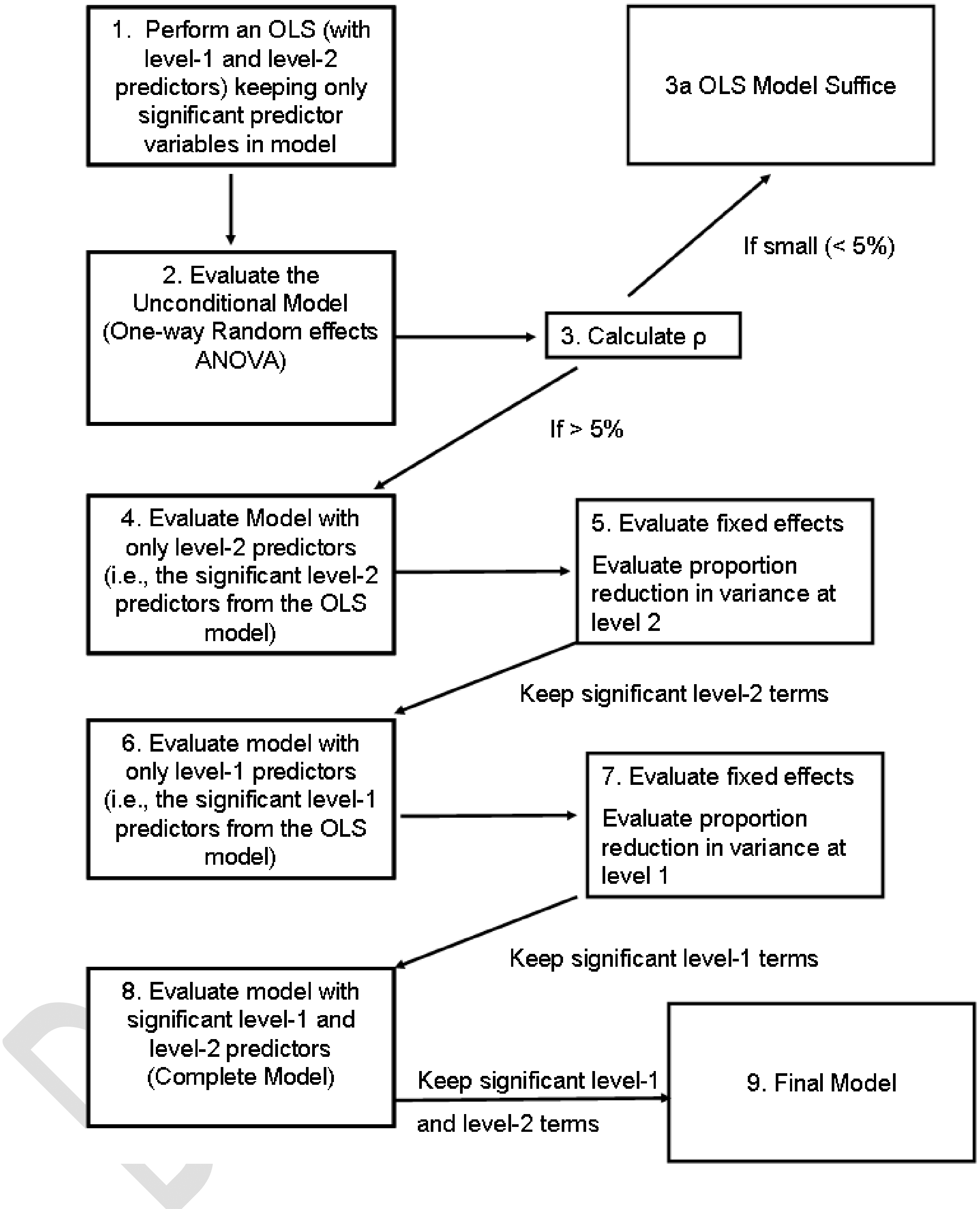
Step 5 evaluates the significance of the fixed effects of the model (i.e., should the level-2 predictor stay in model) and whether the variance at the between cluster units is adequately explained (i.e., is the between cluster units variance, τ_{00}, significant? If not, other level-2 predictor variables might improve the model). The significant level-2 predictors are kept for the complete and final model.

Step 6 is building the random-coefficient model, or developing a model with level-1 predictor variables. The main objective of Step 7 is to evaluate the level-1 predictor (i.e., is the fixed effect significant?), determine whether level-1 predictor varies between institutions (i.e., is the variance of the slope significant? If yes, the variables needs to be modeled as a random effect), and reduce the within cluster unit variance (σ^2). The significant level-1 predictors are kept for the complete and final model.

Step 8 combines the information found in Step 5 and Step 7 to develop the final model. The complete model incorporates the significant level-2 and level-1 predictors and any significant interactions between them. Step 9 evaluates the model by examining measures such as proportion reduction in variance measures and the significance of the fixed effects.

If the model has difficulty in converging to a final estimate, the researcher may choose to change his/her covariance structure from an unstructured to a simple/ diagonal. Other options to help with convergence include centering variables or incorporating group means (contextual model) as a level-2 predictor.

Figure 30: Creating a HLM Model



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APPENDIX 1: Descriptive Statistics of Individual Students

	Mean	**Std. Dev**	**Skewness (Std Error)**	**Kurtosis (Std Error)**
In-class & Out-of Class Activities				
Clarity and Organization	3.1053	.57857	-.243 (0.37)	-.189 (.073)
Collaborative Learning	2.8974	.67479	-.297 (0.37)	-.507 (.073)
Instructor Interaction and Feedback	2.2077	.68620	-.431 (.037)	-.158 (.073)
Program Openness to Ideas and People	2.6243	.86256	.421 (.037)	-.304 (.073)
Program Diversity Climate	4.3311	.58397	-1.439 (.037)	2.742 (.073)
Student Characteristics				
Age	19.11	3.317	4.222 (.037)	22.106 (.073)
Family income	5.0	2.170	.195 (.037)	-.652 (.073)
SAT Verbal	597.98	86.317	.007 (.037)	.044 (.073)
SAT Math	676.62	72.246	-.363 (.037)	.210 (.073)
SAT Overall	1274.601	137.05	-.081 (.037)	.020 (.073)
Overall High School GPA	5.64	.707	-2.430 (.037)	6.839 (.073)

The following are the variable descriptions (Lattuca, Terenzini, Volkwein, 2005).

Student Reports of their In-class and Out-of-class Activities Relevant to Engineering

Clarity and Organization Scale: An individual student's score on a 3-item scale (where 4 = almost always, to 1 = almost never) assessing how often things happened in their classes. Constituent items: "Assignments and class activities were clearly explained;" "Assignments, presentations, and learning activities were clearly related to one another;" "Instructors made clear what was expected of students in the way of activities and effort." (1994 Alpha = .82, 2004 Alpha = .82)

Collaborative Learning Scale: An individual student's score on a 7-item scale (where 4 = almost always, to 1 = almost never) assessing how often things happened in their classes. Constituent items: "I worked cooperatively with other students on course assignments;" "Students taught and learned from each other;" "We worked in groups;" "I discussed ideas with my classmates (individuals or groups);" "I got feedback on my work or ideas from my classmates;" "I interacted with other students in the course outside of class;" "We did things that required students to be active participants in the teaching and learning process." (1994 Alpha = .91, 2004 Alpha = .90)

Instructor Interaction and Feedback Scale: An individual student's score on a 5-item scale (where 4 = almost always, to 1 = almost never) assessing how often things happened in their classes. Constituent items: "Instructors gave me frequent feedback on my work;" "Instructors gave me detailed feedback on my work;" "Instructors guided students' learning activities rather than lecturing or demonstrating the course material;" "I interacted with instructors as part of the course;" "I interacted with instructors outside of class (including office hours, advising, socializing, etc.)." (1994 Alpha = .87, 2004 Alpha = .87)

Program Openness to Ideas and People: An individual student's score on a 4-item scale (where 4 = almost always, to 1 = almost never) assessing how often things happened in the program both in-class and out-of-class. Constituent items: "My engineering courses encouraged me to examine my beliefs and values;" "My engineering courses emphasized tolerance and respect for differences;" "My department emphasizes the importance of diversity in the engineering workplace;" "My engineering friends and I discussed diversity issues." (1994 Alpha = .75, 2004 Alpha = .74)

Program Diversity Climate: An individual student's score on a 4-item scale assessing how often things happened in the program when out-of-class. Constituent items: "In my major, I observed the use of offensive words, behaviors, or gestures directed at students because of their identity" (5 = strongly disagree, to 1 = strongly agree); "I was harassed or hassled by others in my major because of my identity" (5 = strongly disagree, to 1 = strongly agree); "I know some students who feel like they don't fit in this department because of their identity" (5 = strongly disagree, to 1 = strongly agree); "The faculty in my department are committed to treating all students fairly(5 = strongly agree, to 1 = strongly disagree)." (1994 Alpha = .57, 2004 Alpha = .57)

Students' Characteristics

Age: Actual years
Family income: 9-point scale, where 1 = below $20,000 and 9 = more than $150,000
SAT scores: Actual scores on both the math and verbal sections ofthe SATs
Overall high school GPA: 6-point scale, where 1 = below 1.49 (below C-) and 6 = 3.5 to 4.0 (A- to A) *Overall college GPA:* 6-point scale, where 1 = below 1.49 (below C-) and 6 = 3.5 to 4.0 (A- to A)

Characteristic	**Value**	**Count**
Gender	Female	1095
	Male	3366
Transfer status	Transfer	951
	Non-transfer	3510
US citizen	No	514
	Yes	3947
Mothers highest education	High School diploma, GED, or less	1017
	Some College (including Associate's Degree)	1064
	Bachelor's degree	1529
	Advanced degree	851
Father's highest education	High School diploma, GED, or less	852
	Some College (including Associate's Degree)	859
	Bachelor's degree	1436

	Advanced degree	1314
Employment during college	No	1667
	Yes	2794
Months spent in a co-op/intern	None	1803
	1-4	743
	5-8	699
	9-12	595
	More than 12 months	621
Months Spent in a study abroad	None	4010
	1-4	339
	5-8	63
	9-12	34
	More than 12 months	15
Months spent traveling abroad	None	3327
	1-4	941
	5-8	114
	9-12	36
	More than 12 months	43
Months spent participating in a design project	None	2690
	1-4	999
	5-8	318
	9-12	188
	More than 12 months	266
Activeness in a professional society or Engineering	Not at All	1637
	Somewhat	1642
	Moderately	582
	Highly	600

APPENDIX 2: Descriptive Statistics of Organization Characteristics

Institution	Number of Students
Cal State Polytechnic	129
Cal State, Sacramento	46
Case Western	79
Clemson	160
Cornell	144
Embry- Riddle	37
Georgia Tech	138
Howard	21
Iowa State	189
North Carolina AT&T	42
South Dakota	78
Syracuse	44
Ohio State	349
University of Texas Arlington	116
Tri-State	38
UCLA	80
USMA	48
University of Florida	186
University of Illinois, Chicago	136
University of Michigan	182
University of Missouri, Columbia	67
Notre Dame	104
University of Texas, Austin	330
University of the Pacific	14
Western Michigan	67
Worcester	110
Youngstown State	39
Illinois Institute of Technology	114
Lehigh University	121
Princeton	40
University of Arkansas	52
Temple	28
Union College	31
Arizona State	129
Marquette	83
Purdue	278
Virginia Tech	135
Texas A&M	348
MIT	129

Characteristic	Value	Count
Type of Control	Public	24
	Private	15
NSF Coalition Participation	Member of Coalition	15
	Not a coalition member	24
EC2000 review Schedule	Early (1998-2000)	17
	On-time (2001-2003)	14
	Late (2004-2006)	9
Carnegie Classification	Carnegie Research Extensive	27
	Carnegie Research Intensive	3
	Carnegie Masters	5
	Carnegie Bachelors/ Other	4

Characteristic	Mean	Std.Dev	Skewness (Std Error)	Kurtosis (Std Error)
Wealth	78171.77	12500.42	.494 (.378)	1.383 (.741)
Size	2177.59	1819.07	1.104 (.378)	-.009 (.741)

Wealth: Average salary of full professors in engineering
Size: Number of undergraduate engineering degrees awarded in 2004.